A. M. Thesis

MICROWAVE DETECTION OF SHOCK WAVES

Bennett Bovarnick

1949
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Thesis

MICROWAVE DETECTION OF SHOCK WAVES

by

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(B.S., California Institute of Technology, 1946)

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MICROWAVE DETECTION OF SHOCK WAVES

I. Introduction

A. Statement of the Problem.

The Boston University Upper Atmosphere Research Laboratory (BUUARL) is concerned with the investigation of the physical phenomena present in the upper atmosphere of the earth. Experimental evidence is obtained by radio transmission from equipment carried into the upper atmosphere by rockets. The function of the Laboratory is to instrument the rockets and to analyze the transmitted information. The work is coordinated with similar efforts of other laboratories and supported by interested agencies of the United States Government.

A part of the Laboratory's problem included the possibility of determining the velocity of bodies in high speed flight, Mach number greater than unity ($M>1$). Discussions were held during which the measurement of the angle of the shock wave associated with the nose of the body was proposed. The general technique was to be based on the discontinuity of the optical index of refraction occurring at the shock boundary. The boundary was to be considered as the surface of a reflecting medium for electromagnetic waves of radio frequency. However, the exact nature of the experiment was

not settled because of a lack of information available on the amplitude and on the frequency dependence of the reflection which might be obtained from the shock wave.

B. Background.

About this time, a partial solution of the problem of the experimental techniques to be utilized came to the attention of W.C.Moore of the staff of the Laboratory. The results of an experiment "Radar Reflections in the Lower Atmosphere." were reported by A.B.Crawford of the Holmdel Laboratories of the Bell Telephone Laboratories, Inc., at a conference under the sponsorship of the Committee on Electronics of the Research and Development Board. This experiment dealt with the detection of "Angels", sharp echo responses occurring at various altitudes up to 1500 feet, by specially designed radar equipment. The equipment consisted of vertically orientated separate antennas for transmitting and receiving, and operating with interlaced pulses of 3.2 and 1.25 cm. wave-length. The results gave evidence of discontinuities of unknown origin in the lower troposphere with non-determinate characteristics. It seemed that the equipment and results on which the report was made might, with suitable modifications, be used for preliminary exploration of the data necessary for further and

more detailed investigation of the high speed problem.

C. General Plan.

The general plan of the experiment arranged between W.C.Moore of BUUARL and A.B.Crawford of the Bell System was to utilize the equipment, facilities, and assistance of the Holmdel Laboratories of the Bell Telephone Laboratories Inc. in the preliminary investigation of the radio response effects of the high speed projectiles. It was felt that by passing a high speed projectile through the beam, significant information would be obtained about the shock waves associated with the projectile. The problem of determining the proper technique for the beam pass of the projectile was left unsettled at this time.

II. The Experimental Procedure.

A. Equipment.

1. The Radar System.

The radar equipment was the Bell Laboratory vertical incidence system, which had previously been used in some experiments cooperatively performed by the Naval Electronics Laboratory and the Bell Telephone Laboratory. This system was composed of separate antennas consisting of conical horns with vertical orientation, fitted with 30 inch diameter molded polyethylene lenses. The separate antennas are used for transmitting and receiving, thus eliminating the recovery time of a receiver disconnect switch. Since the direct pickup from the transmitting antenna is very low, it is possible to observe reflections from heights as low as the antennas themselves. With this equipment it is possible to make simultaneous observations at two frequencies by transmitting interlaced pulses on frequencies on the order of 10,000 mc. (X-band) and 30,000 mc. (K-band), the wavelengths being 3.2 cm. and 1.25 cm., respectively. The pulse widths are about 0.15 microseconds, with a recurrence rate of 5,000 pulses per second. The transmitted peak power is about 40 Kw. on the X-band and 15 Kw. on the K-band. The half power beam widths are 3° and 1° for the X-and K-bands, respectively. A.B.Crawford, in his article, further reports:[1]

---

1. Bibliography: Reference 1.

> ". . .sensitivity is sufficient to permit detection of a low velocity .22 calibre rifle bullet at its maximum vertical range of 3,500 feet; the reflection from a high velocity .22 calibre bullet can be detected moving in the noise at over 5,000 feet altitude."

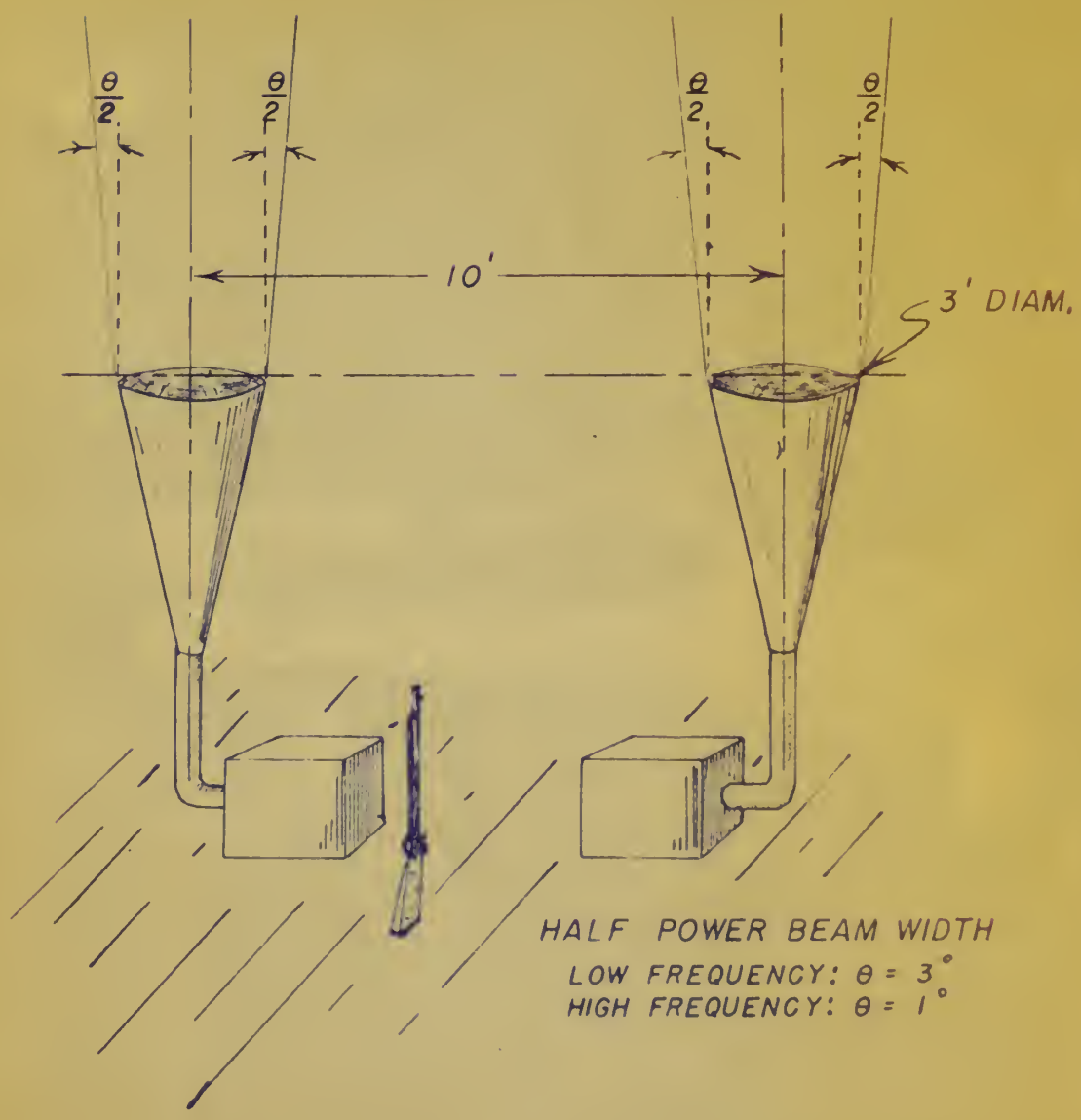
Photographs and diagrams are included showing the arrangement of the antennas and gun. Photographs of reflection traces of typical low and high velocity bullets are also included. The receiver responses were impressed on a double trace tube, the X-band pulses being recorded on the upper trace, and the K-band being recorded on the lower trace. On both traces, time markers were impressed at approximately 500 foot intervals by intensity modulation. Information forwarded from the Holmdel Laboratories indicates that the response of the entire radar system is linear, within the limits of the range of measurement of the experiment.

2. Projectile.

For the projectile, the Boston gunsmiths, Engel and Troesh, supplied the necessary gun and cartridges. Mr. Troesh provided, on a loan basis, a special .22 calibre rifle to be used for firing high velocity bullets which were purchased through their concern. Two types of ammunition were used. One was Type Swift 220 with a listed muzzle velocity of 4,460 feet per second, or with an approximate Mach number of 4. The other was a conventional "long rifle-22"

ARRANGEMENT OF ANTENNAS AND GUN

Fig. 1



ARRANGEMENT OF ANTENNAS AND GUN
FOR MICROWAVE DETECTION OF SHOCK WAVES

Fig. 2

GUN MOUNTING IN FRAME

Fig. 3

with a muzzle velocity of 2,200 feet per second or Mach number of 2. The comparison of sizes for the conventional type "long rifle-22" and Type Swift 220 for projectile and case is shown in the photograph. The base diameter of the projectile is of the order of one-half wave length for the K-band and approximately one-sixth wave length for the X-band. Preliminary firing was to have been carried out with the long rifle cartridge to determine optimum alignment and projectile path.

3. Data Recorder.

In order to have a complete record of the events as pictured on the oscilloscope screen, a motion picture camera was obtained on a loan basis from Jack Seltzer Co. The camera was given preliminary tests by J. Seltzer and W.C.Moore in order to determine the optimum conditions for recording the oscilloscope picture. The camera agreed on was a 16mm. Bolex, using a telephoto lens with the lens stopped at f:2.0, and focused for an object distance of 2 feet. The camera speed used was 16 frames per second.

Fig. 6

	MUZZLE VELOCITY	MACH NO.
HIGH VELOCITY	4460 Ft./Sec.	4
LONG RIFLE	2200 Ft./Sec.	2

COMPARISON OF HIGH VELOCITY BULLET WITH LONG RIFLE BULLET

B. Methods.

Various techniques were proposed in order to obtain reflections from the bullet and associated shock wave successfully.

1. Oblique Orientation.

The first method provided that the bullets of both the long rifle and high velocity cartridges be fired at right angles to the beam and approximately through the axis. To do this the beam was directed at an angle of 30° with the horizontal, while the gun was mounted on a roof top at a height of approximately 15 feet. Results of this technique were negative for both types, although the receiver did pick up some ground clutter reflections, better known as "Angels."

2. Beam Split.

For the next attempt, the beams were vertically orientated. The bullets were fired horizontally between the beams at a height of approximately 10 feet above the lenses. Again, the results obtained were negative for both types.

3. Beam Tunnel.

A slight modification was made for the following attempt. The path of the bullet, still horizontal, was through the transmitter and receiver beams, respectively. This time, only the "long rifle" bullets were fired. Once more the results were negative.

4. Vertical Orientation.

A new set-up was tried next. The antennas were still vertically orientated. Now, however, the gun mount was fixed to the antenna carriage so that the rifle axis was parallel to the beam axis. Again only long rifle bullets were fired. Only the K-band trace was responding. This time, for the first time, positive results were obtained. By visual observation the response on the scope followed what seemed to be an exponential decay to approximately 3,000 feet. It is this technique which was used throughout the remainder of the experiment.

5. Suggested Attempt.

An effort might have been made to give the entire apparatus a horizontal orientation. This could have been done by rotating the mount 90° from the vertical. In this way, the set-up is substantially the same as in attempt number 4. But it was felt that the danger from stray bullets prevented an application. The advantage, of course, would have been to eliminate gravitational retardation of the projectile.

C. Data.

1. Firing.

Once the proper set-up for the radar system and gun mount had been determined, the experiment was

reduced to a mechanical procedure of firing the bullets in the desired succession.

The bullets were fired in the following sequences, with varying time intervals between successive shots:

a series of 12 long rifle shots;

a series of 13 high velocity shots;

a series of 4 long rifle shots.

In each series, some shots were fired as single shots, consecutive shots not being fired until the reflection response of the previous shot had faded out. Others were sent up in a rapid-fire sequence. The only effect of the rapid-fire sequence was a crowding of the response traces, since two and occasionally three reflections were present at the same time. In order to fire the high velocities from the long rifle position, slight modifications were required.

2. Firing Log.

The log of the film record of the firing is as follows (with titles):

from to (length of activity in feet of film)

a. 0- 13½

Long Rifle through Beam

b. 13½- 15

High Velocity between Vertical Beams

(no pictures taken)

from to (length of activity in feet of film)

c. 15- 17½

Long Rifle Vertical between Beams

Markers 500 ft.

Top X-band

Bottom K-band

d. 17½- 41½

24 feet of long rifle bullets.

Series shot in sequence (12 shots).

e. 41½- 53

High Velocity Vertical

Went full scale (4,000 feet).

Same sensitivity including long rifle

10 shots.

f. 53- 55

Blank of 2 feet - end.

g. 55- 61

Gun mount shifted

h. at 61

Check gun alignment

5,000 feet - full scale on X-band (top).

i. 61- 65½

Off scale on K-band (5,000 feet).

Out at 4,000 on X-band

j. 65½- 69

Ditto.

from to (length of activity in feet of film)

k. 69- 77

Sequence of shots - high velocity, 6,000 end of scale on K (lower trace) (shifted scale) end 77 feet.

l. 77- 80

Single shot - high velocity, 6,000 on right hand end of lower scale.

m. 80- 87

Long rifle. To end of film.

III. Transcription of Experimental Data.

A. Technique.

The next step in the experiment was to calibrate the test film. In general, the procedure would be to determine the velocity and response amplitude variations by direct readings from the film, and from these direct readings to determine the differences for successive frames. Since the camera speed was known to be 16 frames per second, it is possible to obtain both the response amplitude, and altitude and velocity as time dependent relations, and permits correlation of the resulting information.

B. Equipment and Accessories.

1. Projector.

In order to obtain more accurate readings from the film, the images were amplified by using a Navy Film Scoring Viewer, Mark 1, manufactured by Bell and Howell Co. for the U. S. Navy, Bureau of Ordnance. The Film Scorer light path includes a three-fold reflection between the source and final image from mirrors at 45° with the axis of the light beam. The image appears on a ground glass screen approximately 15" by 15", which is an integral part of the projector.

2. Counting Device.

In order to facilitate the proper

identification of the frames and to simplify the determination of the time duration of the response due to a single projectile, a small counter was mounted on the projector. The counter was adapted from a bicycle cyclometer manufactured by the Veeder Mfg. Co. of Hartford, Conn. The cyclometer was gear driven from the main drive sprocket in an 8-10 ratio so that each frame in a sequence would be specified by its own number. Thus the number of frames included in a single sequence might be determined as the direct difference of the initial and final reference numbers.

3. Film Marker.

Further, it was found convenient to mark the film strip so that a sequence for a particular shot might be identified with a minimum of effort. This was done by punching the edges of the film strip adjacent to the initial frames of each sequence. For a series, the punches were cumulative. Thus, for the series of 12 long rifle shots, the punches increase numerically from 1 to 12. The other series are similarly marked. For each sequence the initial punch mark of the identification is adjacent to the initial frame of the sequence, and successive marks are adjacent to successive consecutive frames.

4. Image Scale.

Two different scales were used while

transcribing the data, each associated with a different projected image. When the image was viewed on the ground glass screen, the scale used was a reticule with a single set of rectangular axes. This image was superposed on that of the film and readings were taken with respect to the unit divisions on the reticule. The scale factor was 170 feet-1 unit. However, a larger image was obtained by projecting the image on a wall about 5 feet away. Although using the wall projection was somewhat less convenient, the advantages obtained--a clearer image, and sharper contrast with the background--more than offset the inconvenience. Further, readings with finer scale divisions were obtained by placing a sheet of 10 square to the one-half inch cross section paper in the image plane. The new scale, which was possible only because of the increased contrast with the background, increased the accuracy of each reading. The scale factor in this case was approximately 190 feet - 1 inch.

C. Comment.

The data recorded from the projected film consists of direct readings only. These are respectively:

1.) frame number in the sequence.

2.) length of travel of the response peak along the trace.

3.) amplitude of the response peaks above their

traces for the X-band and K-band respectively.

The time required to collect this information from the film amounted to approximately two hours for each sequence of a shot.

IV. Analysis.

A. Proposed technique.

During the preliminary stages of the analysis, a paper presented at the 1948 Annual Meeting of the American Physical Society[2] provided some helpful information; namely, that the reflectivity of shock fronts is extremely small, of the order of 10^{-5}. This low value nullified any attempt that might be made to determine the contribution of the shock front to the response amplitude of the receiver. With this in mind, the technique took on the nature of a comparison. The comparison was to be between the actual response amplitude for all velocities, and a predicted value using the range for Mach < 1 as a base. The underlying principle to the calculations is that the response is proportional to the inverse fourth power of the target distance. Preliminary calculations indicated that this relation is valid only in the range of target velocities $M<1$, and falls off rapidly in the region $M>1$. The calculations may be expressed in the following symbolic fashion.

1. Using R_a and d^4 ---- R_o $(M<1)$ from records

---

2. Bibliography: Reference 2.

2. Using R_o and d^4 ---- R_p $(M > 1)$

3. $v = f(d)$ from records

4. $R_p = f(R_o, d^4)$

$= f(R_o, v)$

5. Now $\frac{R_a}{R_p} = f(M) = f(v)$

where:

d - altitude

v - velocity

M - Mach number

R_a- actual response amplitude

R_p- predicted response amplitude

R_o- constant of proportionality

In words, these relations say that if the inverse fourth power variation is accepted in the range $M < 1$, then with the actual response taken from the data and the corresponding altitudes, it should be possible to determine the constant of proportionality - R_o. Having evaluated this constant, it should now be possible to determine what the magnitude of the reflection response amplitude would be for the range $M > 1$, if the inverse fourth power law holds. These values may then be compared with the actual response as taken from the film. The next stage is to determine the velocity. This could be done by taking directly the increments in altitude, and evaluating

the ratio of altitude increment to the constant time increment of one-sixteenth of a second. Thus, the response, actual and predicted, may be plotted against the velocity and then the effective reflection area, which is merely the ratio of the predicted response to the actual response, may also be plotted against velocity. From these plots, which are at best only good approximations to the real functions and curves of the situation, it is possible to make comparisons, and thus determine the contributions to and effects on the response due to the shock front. Since the base diameter of the bullet is of the order of one-half wave-length for the K-band, and one-sixth wave-length for the X-band, it is possible that the bullet may act as a scattering particle. In this case, the intensity of its own radiation field would vary as the fourth power of the distance from the bullet.[3] Thus, the reflection response amplitude of the radar beam would vary as the inverse sixth power of the altitude as a scatterer, instead of, as has been assumed, the inverse fourth power as a reflector. But, since the bullet is of an intermediate order of magnitude, not large compared to the wave lengths as would be required for a reflector, and not small compared to the wave lengths as would be required for a scattering particle, then either phenomena may be accepted as an initial assumption.

---

3. Bibliography: Reference 3.

If the values obtained from this assumption are divergent from the measured values, then, of course, the other must be tried. In either case, the analytical procedure would be similar, so that the method outlined above should be sufficient.

B. Velocity Variation with Altitude.

1. Mechanical Increments.

a. Dependence on a third parameter - time.

It is possible to utilize information on the recording film to determine directly the velocity of the bullet as an average over the interval between frames. Since even this is slightly discontinuous, it is more advantageous to take an average of the changes in displacement (which is what is actually measured) over each pair of consecutive frames for a number of different bullets. Doing this, of necessity, means that two assumptions are accepted: 1. that the motion and velocity of the bullets are approximately uniform; and 2. that the initial frame of each sequence is neglected, since the camera picks up the bullet at a slightly different time after its being fired from the gun, at heights varying from 116 feet to 330 feet. Past experience, and information from gunsmiths is assurance that the first assumption is justifiable. The second assumption is justified by the data itself, since these initial variations are rapidly averaged

out, and later frames become indistinguishable, at least so far as displacement is concerned. The result of the step is shown as a plot of the average velocity versus frame number, where frame number is a time scale whose units are equivalent to multiples of one-sixteenth second.

b. Altitude dependence on time.

It is also necessary to determine the average displacement with respect to time. This is the most direct information that may be obtained from the data, since the data transcribed are position, frame number (elapsed time), and response amplitude. Again, a plot is made using frame number (time) as the independent parameter and displacement or altitude as the ordinate. The plot of the average displacement versus frame number is also shown.

c. Elimination of parameter.

Now, by combining the two previous plots, displacement versus frame number and velocity versus frame number, the independent parameter frame number (time) is eliminated, and, in this fashion, a plot of velocity versus displacement is determined. This plot, too, is shown. When this information has been tabulated, the transformation from the functional dependence of response amplitude on altitude to the dependence on velocity or the equivalent dimensionless Mach numbers can be started.

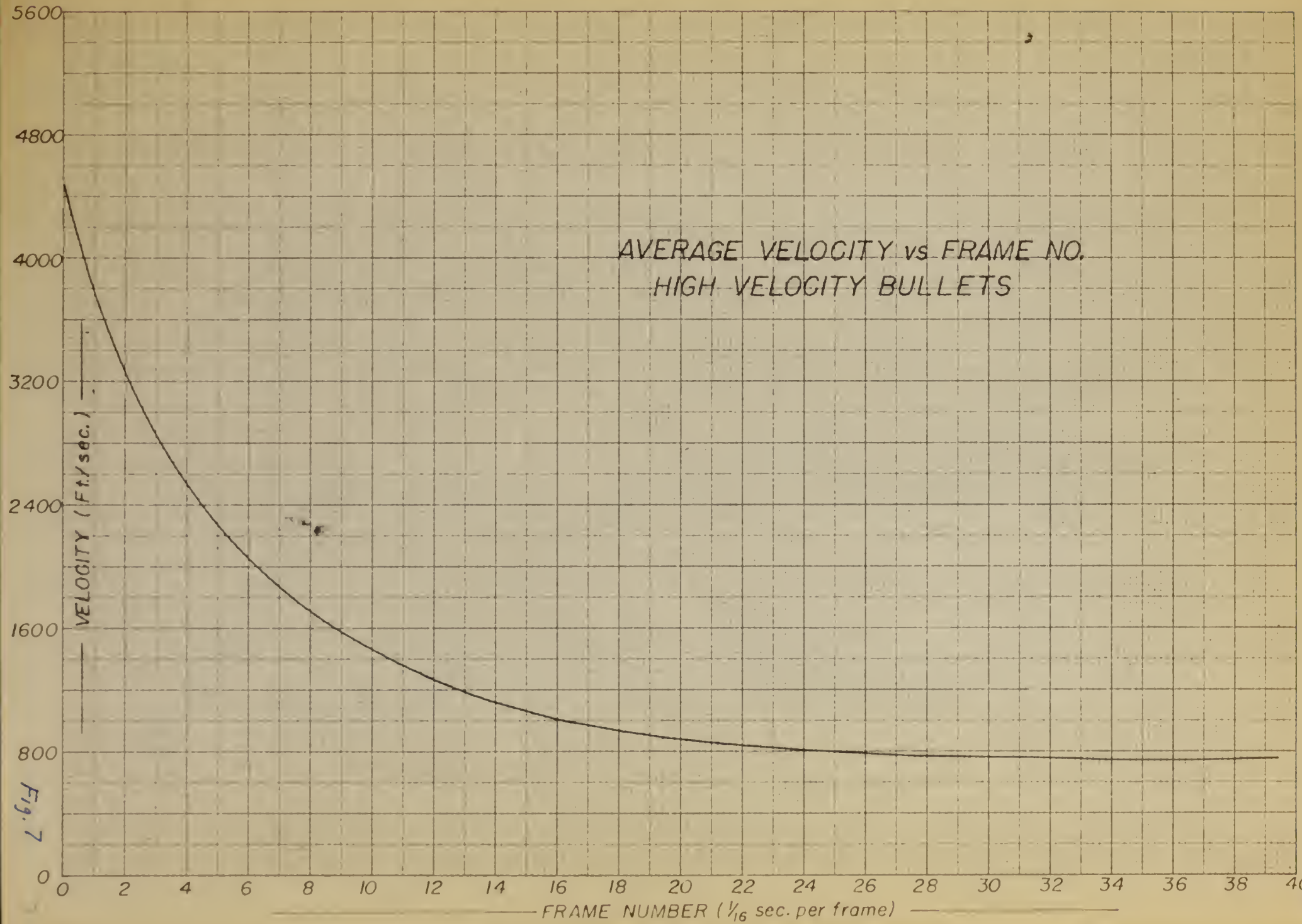

AVERAGE VELOCITY vs FRAME NO.
HIGH VELOCITY BULLETS
VELOCITY (Ft./sec.)
FRAME NUMBER (1/16 sec. per frame)
5600
4800
4000
3200
2400
1600
800
0
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40

Fig. 7

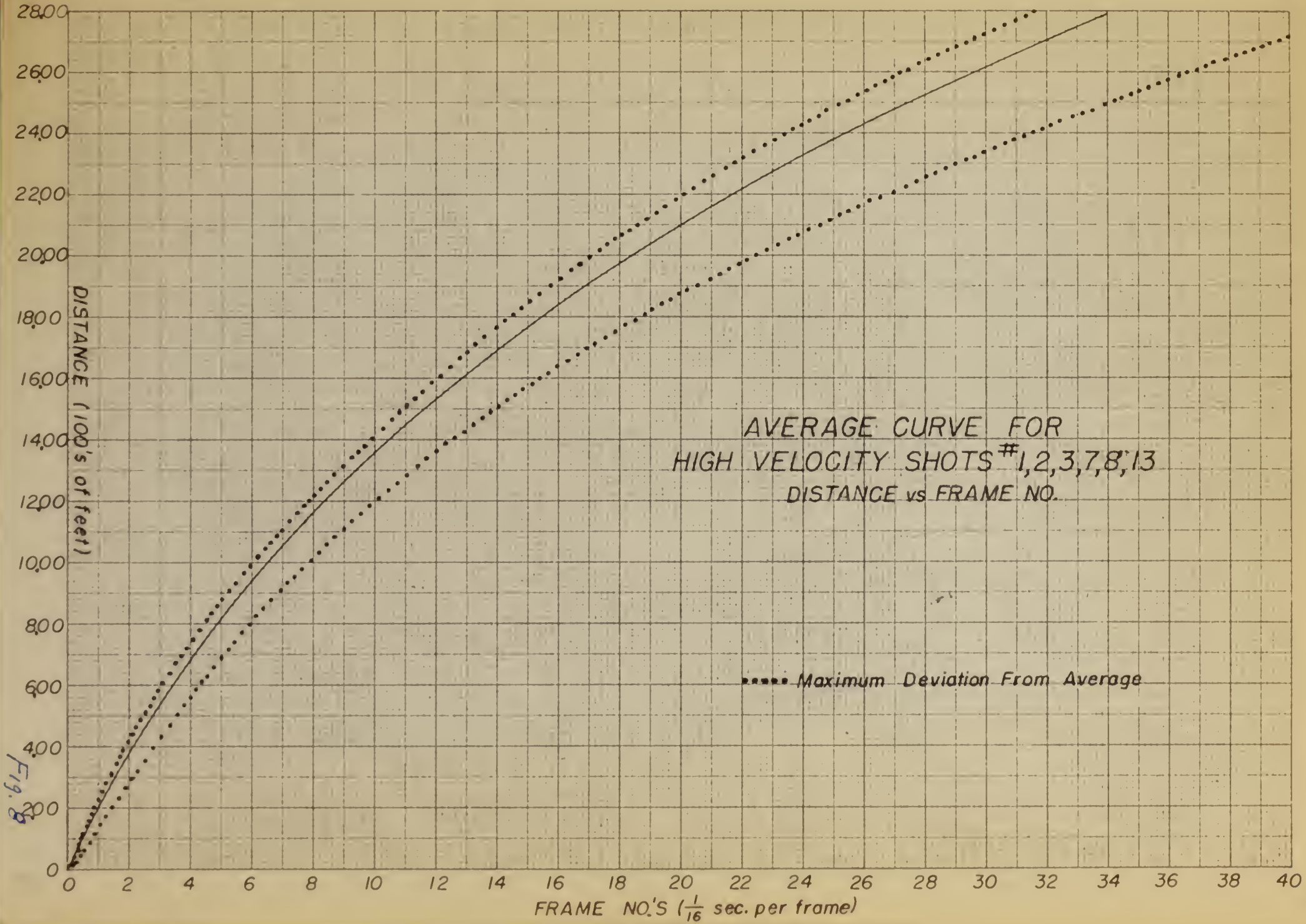

AVERAGE CURVE FOR
HIGH VELOCITY SHOTS #1,2,3,7,8,13
DISTANCE vs FRAME NO.
Maximum Deviation From Average
DISTANCE (100's of feet)
FRAME NO.'S ($\frac{1}{16}$ sec. per frame)
0
2.00
4.00
6.00
8.00
10.00
12.00
14.00
16.00
18.00
20.00
22.00
24.00
26.00
28.00
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40

Fig. 8

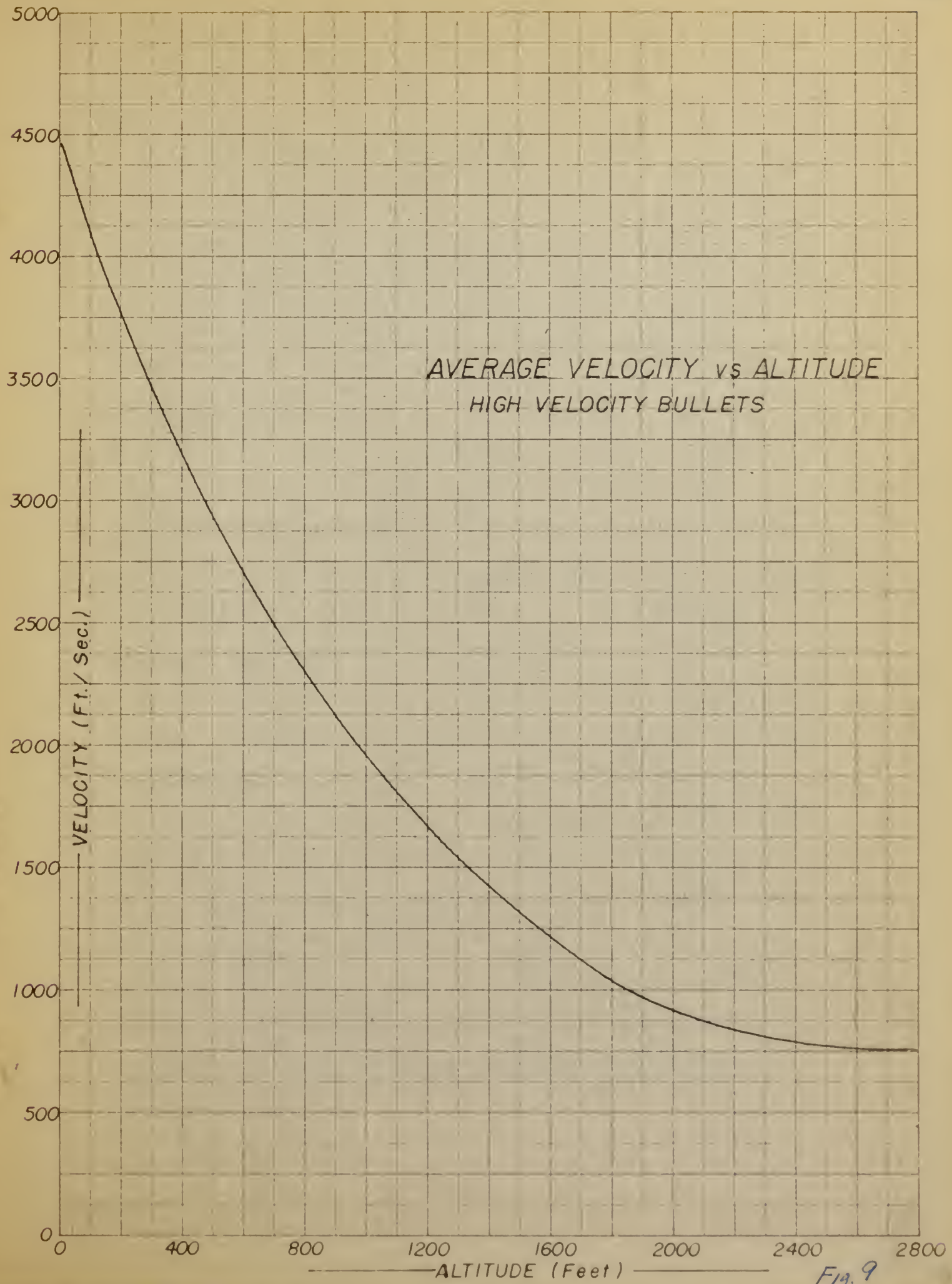

AVERAGE VELOCITY vs ALTITUDE
HIGH VELOCITY BULLETS
VELOCITY (Ft./Sec.)
ALTITUDE (Feet)
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
0
400
800
1200
1600
2000
2400
2800

Fig. 9

2. Theoretical Derivation.

It was also felt that it would be advisable, if it were at all possible, to determine a purely algebraic expression for the relation between response and velocity. It was realized that excessive calculation might be required in order to evaluate the numerical constants that would unavoidably occur, and for that reason this side of the analysis was not pursued too actively. At any rate, a beginning was made in this direction. This consisted of the solution of the well-known problem of a projectile moving vertically upward against the retardation of gravity and resistance of the air. The information available indicated that the time rate of decrease of velocity with respect to air resistance would be dependent upon the square of the velocity itself. This was the same as the opinion of Dr. Georg Joos of Boston University. He holds that except for the lower velocities, neglected in this experiment, and in the region of Mach equal to unity, where variations would be expected to occur, the air resistance has been found to vary quadratically with the velocity. However, in the absence of more definite information for this particular problem, the solution was attacked from three separate differential equations. These equations are:

(1) $\ddot{x} = -g - a\dot{x}$

(2.) $\ddot{x} = -g - a\dot{x} - b\dot{x}^2$

(3.) $\ddot{x} = -g - b\dot{x}^2$

where:

$\ddot{x}$ represents acceleration

$\dot{x}$ represents velocity

g represents gravitational force constant of 32.2 feet per second per second

$\left.\begin{matrix} a \\ b \end{matrix}\right\}$ represent constant coeficients of force of air resistance

To the algebraic solutions of these differential equations, there is only one known condition that may be applied. This is the initial condition of the projectile motion; i.e.: at time t = 0, the projectile is starting from an initial position of zero altitude (x = 0), with a velocity of 4,460 feet per second (or within the limits of the accuracy of the data, Mach number of 4.0). The three differential equations lead to the following solutions, which can be developed without much difficulty:

$$(1') \quad x = \frac{1}{a}(v_o - v) - \frac{g}{a^2} \ln \frac{av_o + g}{av + g}$$

$$(2') \quad x = \frac{a}{2s^2 b}\left[\left(1 - \frac{s}{a}\right)\left(\ln \frac{2bv + a + s}{2bv + a - s} - \ln \frac{2bv_o + a + s}{2bv_o + a - s}\right) - 2\ln \frac{2bv_o + a + s}{2bv + a - s}\right]$$

where $s^2 = a^2 - 4bg$

$$(3') \qquad x = \frac{1}{2b} \ln \frac{g + bv_o^2}{g + bv^2}$$

where $\ln = \log_e \quad v_o = v\big|_{t=0}$

When these solutions had been evaluated numerically, it was found that only

$$(3') \qquad x = \frac{1}{2b} \ln \frac{g + bv_o^2}{g + bv^2}$$

gave satisfactory results. The remainder of the problem, was the evaluation of the constant coefficient b. This was done by making several curves of the form of equation (3') for several arbitrary values of b, within the range indicated by a fast numerical approximation. The value indicated by this approximation is one of the order of 10^{-4} with dimensions of reciprocal length. Now, it became possible to determine a better value by trying to match the curve of equation (3') to the average curve of velocity versus altitude. It is possible to do this since an end point on both curves represents the same initial condition. This step leads to a value of 7×10^{-4} for b. This was more accurately determined by a point by point comparison with the average curve values of b on either side of 7×10^{-4}. The final value determined was 8.2×10^{-4}. It was felt advisable that the derivation be extended and an expression in terms of Mach number be obtained. The result of this step, a direct substitution, is

$$(3'') \qquad M = \left[e^{-2bx} \left(\frac{g}{bv_c^2} + M_o^2 \right) - \frac{g}{bv_c^2} \right]^{\frac{1}{2}}$$

where v_c - acoustic velocity

$\approx$ 1,100 feet per second

and $M_o = -\frac{v_o}{v_c} \approx 4$

C. Variation of Reflection Response Amplitude.

1. By Direct Measurement.

a. Altitude dependence.

The amplitude of the reflection response was also some of the direct data transcribed from the film. Since the X-band and K-band responses are independent, it is possible to treat them almost simultaneously for similar procedures. Further, both amplitudes have been measured in arbitrary scale units. Since we are to determine a ratio of the actual response to a predicted response, there is no need of evaluating the amplitude units in terms of energy. Instead, it is possible to work directly with the values recorded in the data. As a beginning, the tabulated data of response amplitude was plotted against altitude for X-band and K-band, and average curves obtained. These curves, which are approximately rectangular hyperbolas, are shown.

b. Altitude dependence on time.

Fig. 10

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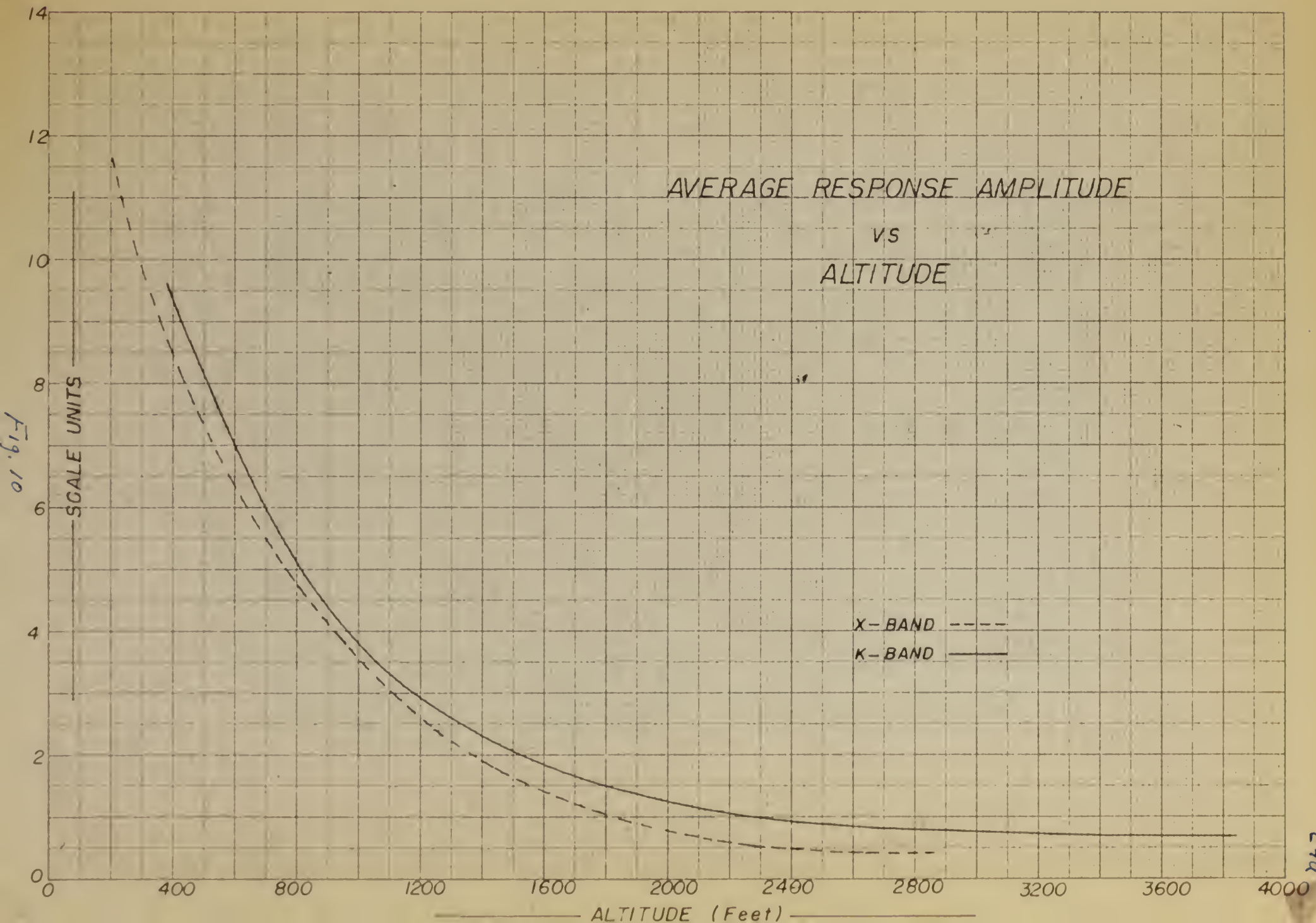

Fig. 10

The next stage consisted of eliminating the distance parameter in an effort to determine a significant response versus velocity relation. This was done by making a cross-plot of response versus distance and velocity versus distance curves. The resulting curves, as shown, indicate that the relations for the X- and K-bands are very much alike, even though the K-band has an almost constant higher measured response than the X-band. These curves, although smooth, are not of a readily recognizable functional form.

2. Inverse Fourth Power Predictions.

a. Altitude dependence.

The analysis was now at the stage where simple theoretical considerations might be applied and checked against measured experimental data. Some intuitive opinion supports the hypothesis that the reflected response would vary as the inverse fourth power of source-target distance. This is based on the theory that the strength of a radiation pattern varies inversely as the square of the distance from the source. Accepting that the target acts as a secondary source, an inverse square law is impressed on an inverse square law magnitude and thus the result

(1) $$\frac{1}{d^2}\left(\frac{1}{d^2}\right) \rightarrow \frac{1}{d^4}$$

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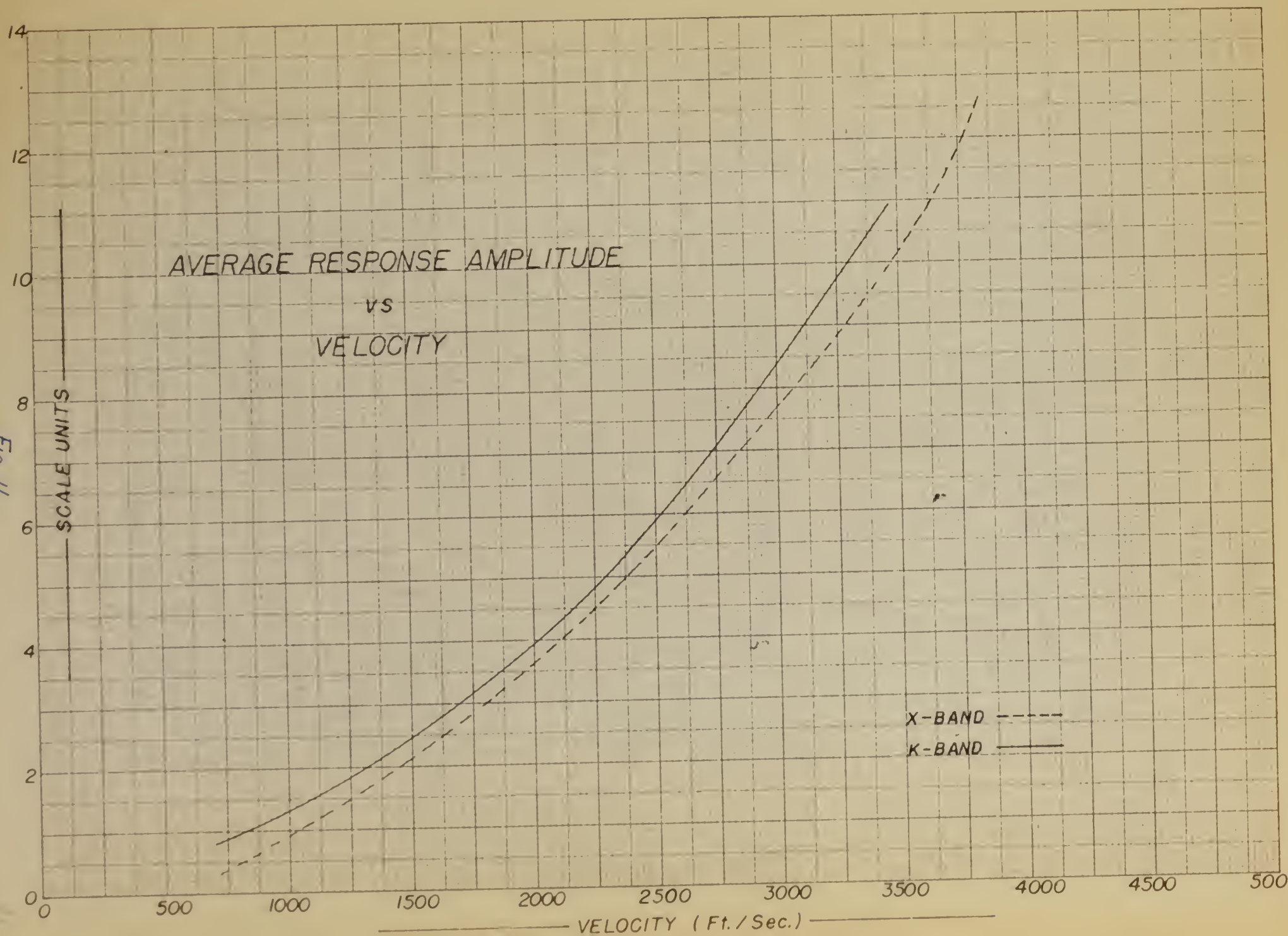

Fig. 11

leads directly to the inverse fourth power hypothesis. In this way, a basis is accepted for a first approximation. It is now possible to begin determining the constants of proportionality as in relation (L) of section IV A. Here for a group of values in the upper limits of the range of the bullets, it is possible to determine an approximate average value of R_o by using the relation

$$\frac{\sum^{n} R_a d^4}{n} = R_o$$

Depending upon the particular shot, the value of n varied from 7 to 14. The limits used on the particular products $R_a d^4$ were arbitrarily chosen, only two conditions being applied. One of these is that the altitude be such that the velocity is less than Mach equal to unity. The other that the values of the products for a single shot be reasonably consistent. It so happens, as is readily apparent from the data, that the first condition is implicitly contained when the second is satisfied. This is really the first time during the analysis that any exceptional significance may be attached to the critical velocity condition of Mach equal to unity. Now that R_o has been determined, predictions of response amplitude could be made on the strength of the inverse fourth power assumption. This was done, and results of response versus altitude were plotted for comparison with the

recorded data. Although a very sharp divergence occurs between the two curves, predicted and actual, this divergence is difficult to correlate with previous data. In an effort to obtain a clearer view of the connecting relation, the plots were made on log-log paper (2 x 3 cycles). A sample of these plots containing the X-band response for 3 shots is shown. These log-log plots are very helpful because the information contained in them is more readily apparent. The effects at the critical velocity are more pronounced. It should be explained that in the log-log plots the straight line deviation from the fourth power straight line is not quite true, but is shown only as a first approximation. More exactly the actual response has a curvature to it, and in fact has a seemingly non-periodic oscillation. Some efforts were made to determine the first few terms of the series expansion describing this curvature in terms of descending powers of the altitude. But the limited accuracy of the data narrows the success which may be obtained. Nevertheless, a system of eight simultaneous equations in the eight unknown coefficients of the terms of the expansion was set up. The work of evaluating this system was held to be unprofitable and therefore was not carried to a conclusion. This is one possible point of departure for those who carry on further studies of this problem.

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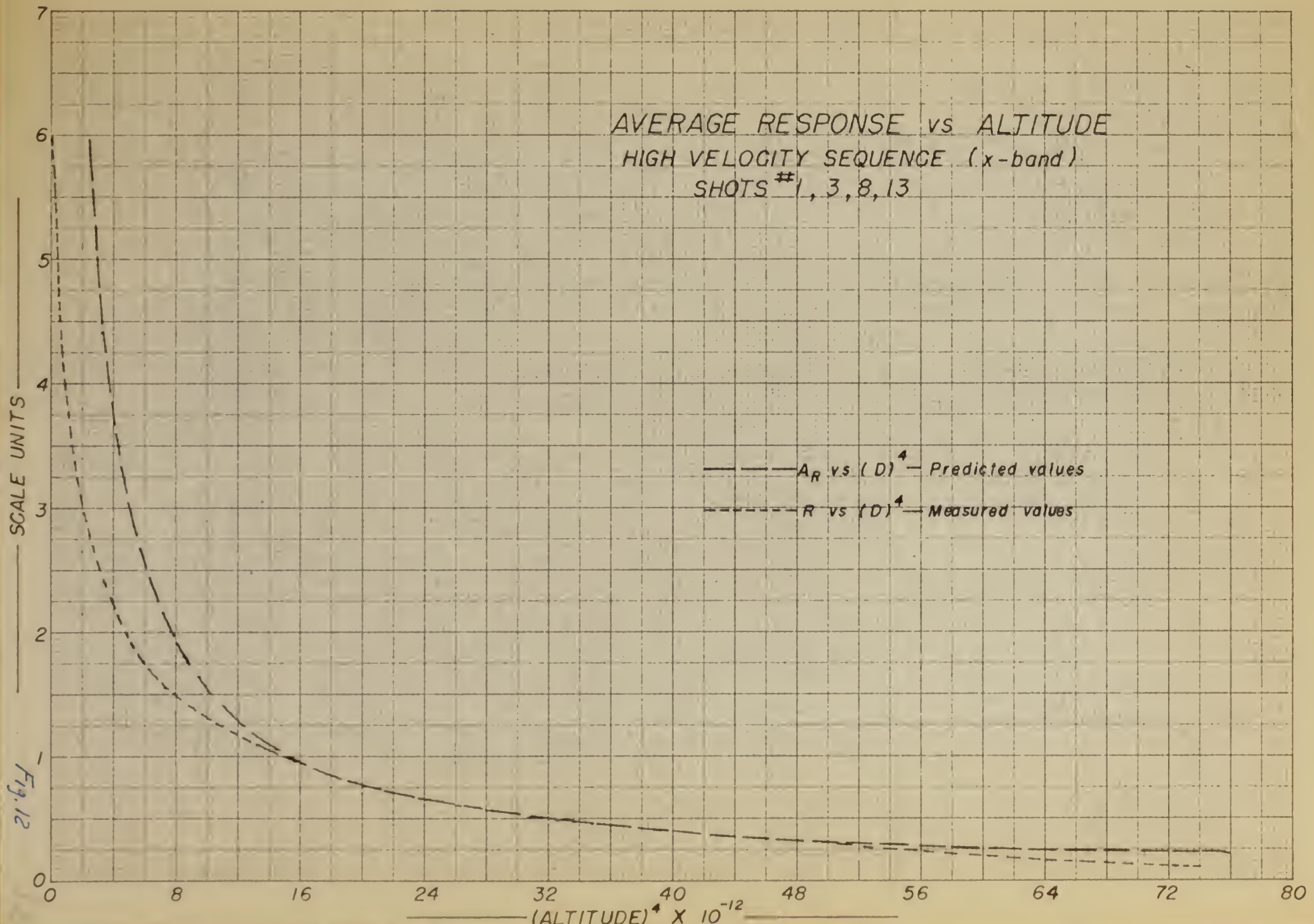
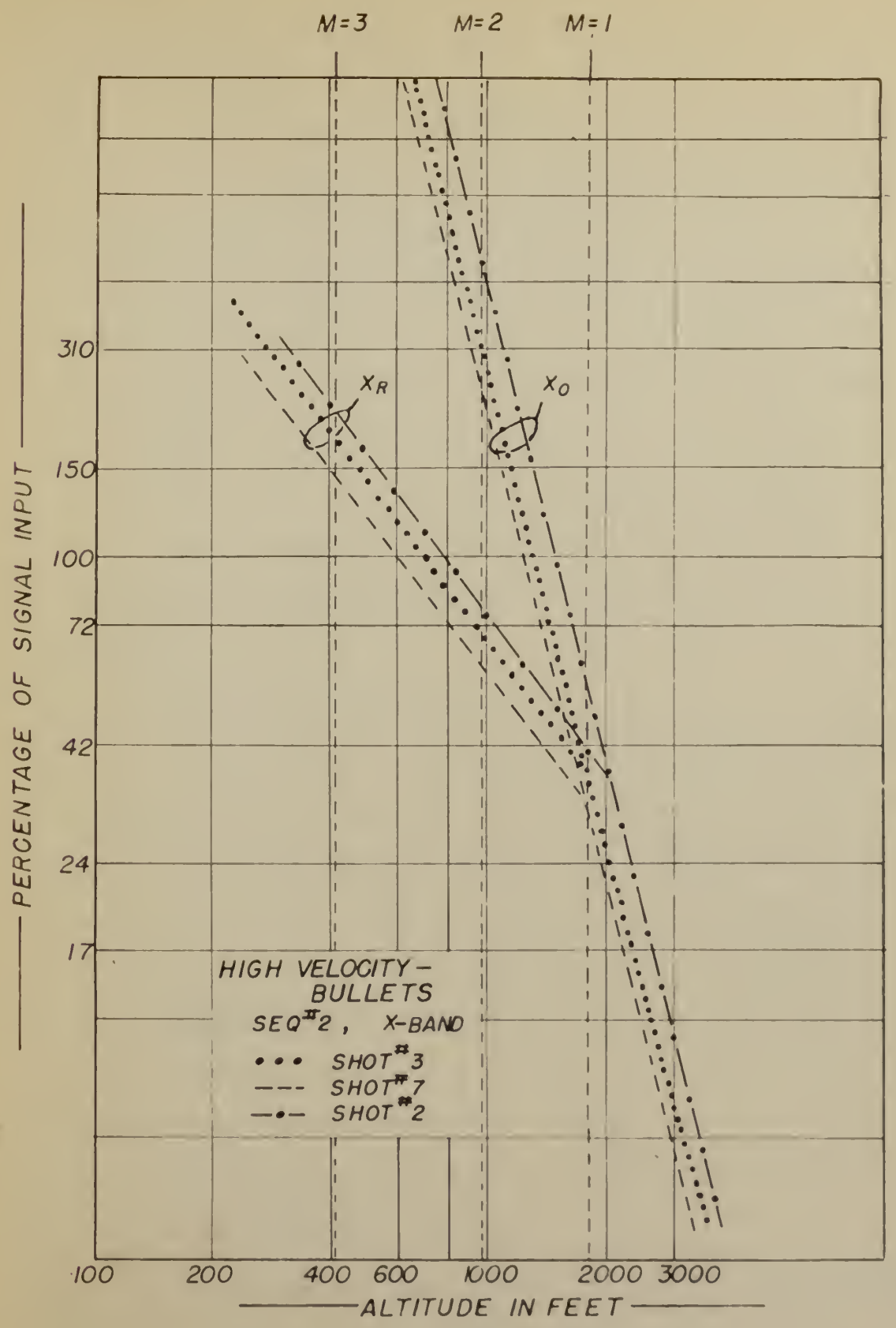


Fig. 12



ALTITUDE vs PERCENTAGE OF SIGNAL INPUT
(MICROWAVE DETECTION OF SHOCK WAVES)

Fig. 13

b. Velocity dependence.

Because of the exponential nature of the velocity-altitude relation, equation (3''), no efforts were made to calculate values of response as a function of velocity that might be expected. Although the analytical plan proposed in section IV A was on the general side, it has been followed fairly closely to this point. The only blank space is for step 4, eliminated because it would require excessive expenditure of effort, and because it is not, in itself, of prime importance to the problem.

D. Effective Reflection.

It is possible to proceed directly to the final stage of the analysis, the determination of an effective measure of reflection response as it depends on velocity. Because no direct response velocity relation had been determined, it was again convenient to carry out the work in terms of the independent distance parameter, and then combine the separate results. The velocity-distance relation has been plotted and defined, and so the only difficulty remaining is that of defining an acceptable measure of reflection response. It would be inadvisable to use a direct system, because that would be dependent upon the constants and limits of the radar system, the target size, and possible other indeterminate quantities.

Instead a ratio measure was decided upon. It was felt that this would eliminate dependence upon the system and target. As a base, the predicted values of the inverse fourth power were used and the recorded response compared with it. The argument in favor of this type of ratio is that the predicted values are taken from the same system as the recorded values, and thus the resulting ratio is very likely to be independent of the system parameters for a particular frequency. Accordingly, the ratio, called the effective reflection area or ratio, was calculated as has been already indicated and curves of the ratio versus distance were plotted. The curves of the ratio versus velocity, expressed in terms of Mach number, were thus determined. These curves are of especial interest, for their general shape is independent of the frequency. These curves are shown both for single shots, in terms of K-band and X-band frequencies, and as cumulative averages, so that smoother composite curves might be obtained.

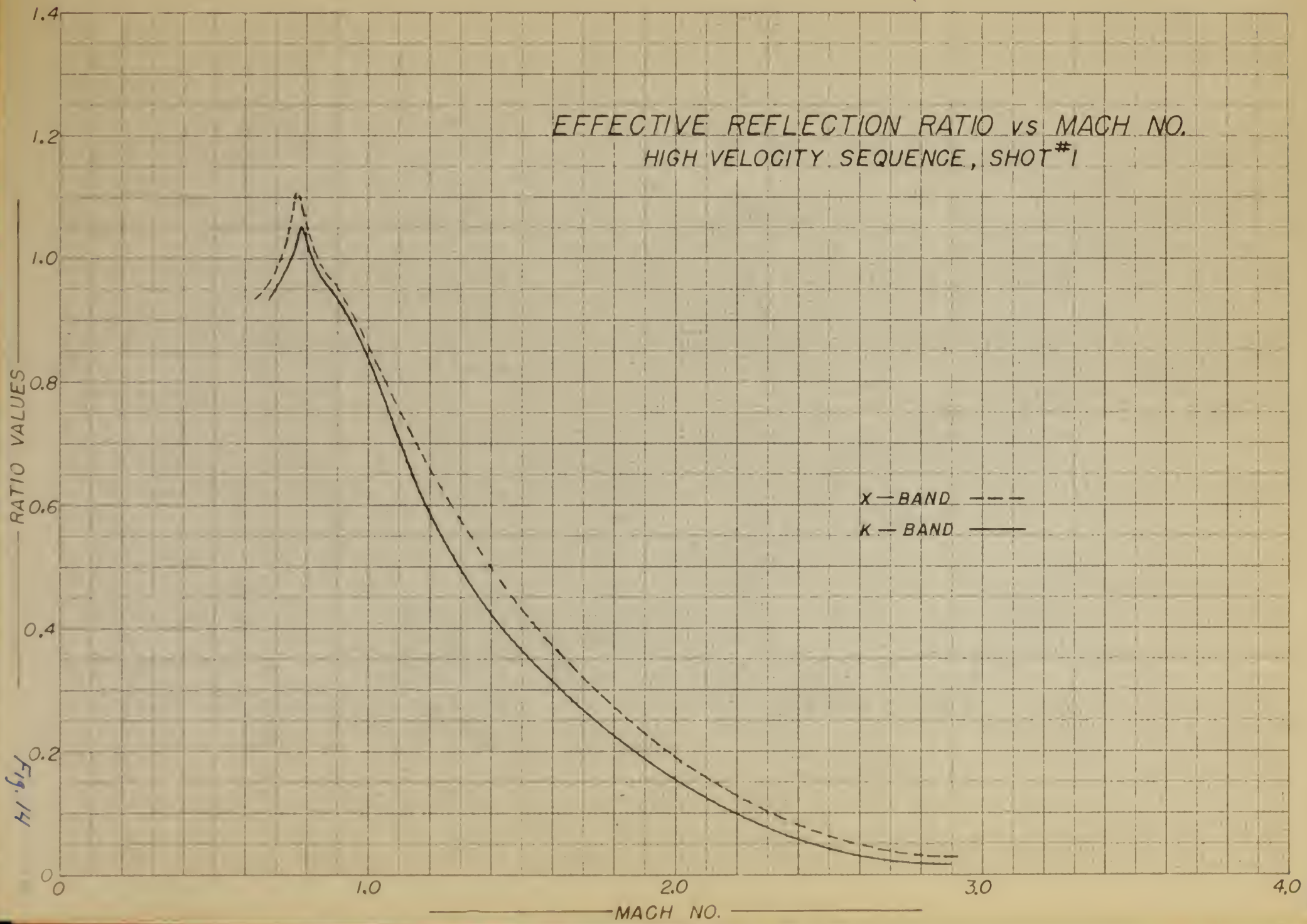

EFFECTIVE REFLECTION RATIO vs MACH NO.
HIGH VELOCITY SEQUENCE, SHOT #1
X — BAND
K — BAND
RATIO VALUES
MACH NO.
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0
1.0
2.0
3.0
4.0

Fig. 14

29a

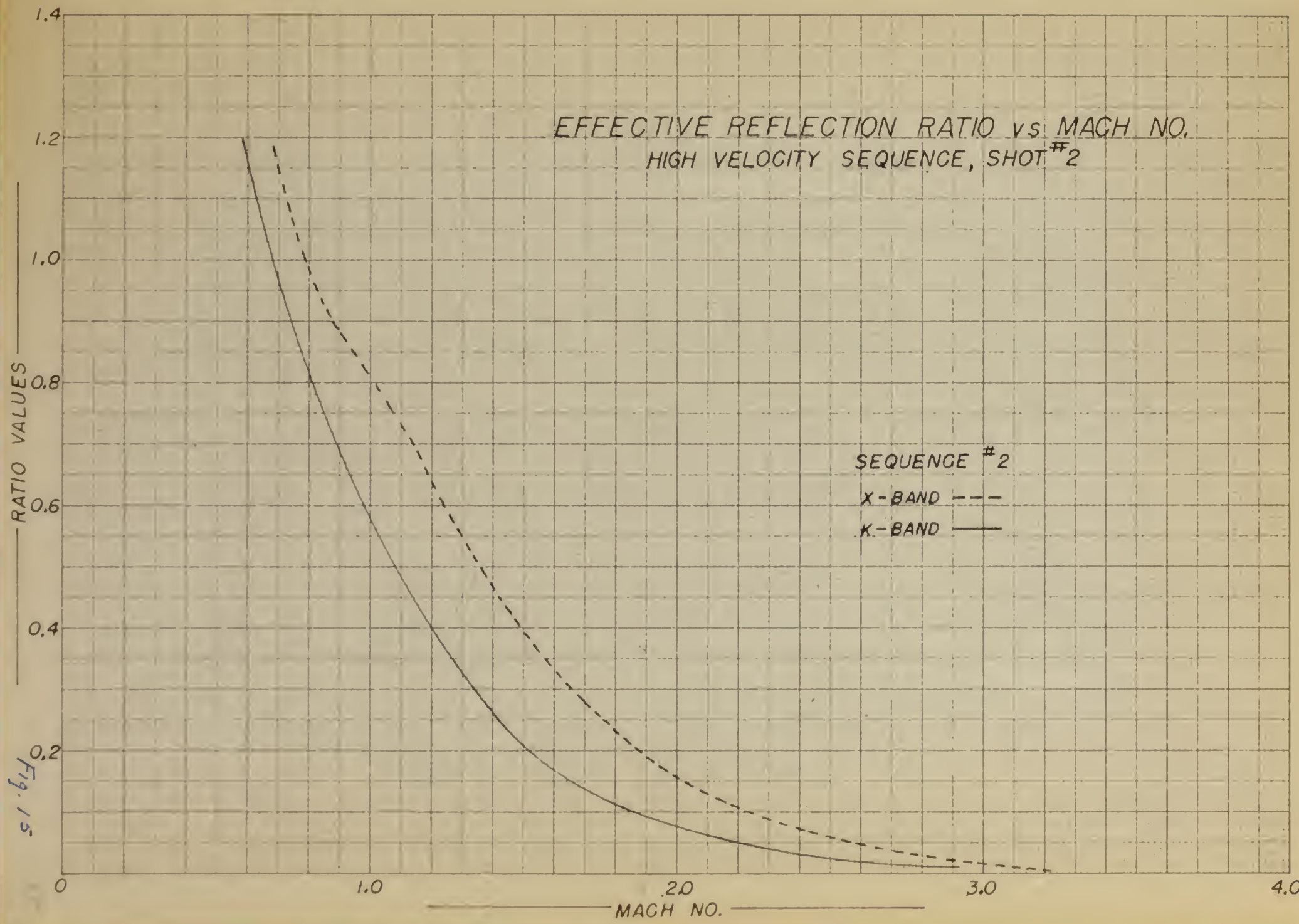

EFFECTIVE REFLECTION RATIO vs MACH NO.
HIGH VELOCITY SEQUENCE, SHOT #2
SEQUENCE #2
X-BAND
K-BAND
RATIO VALUES
MACH NO.
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
1.0
2.0
3.0
4.0

Fig. 15

296

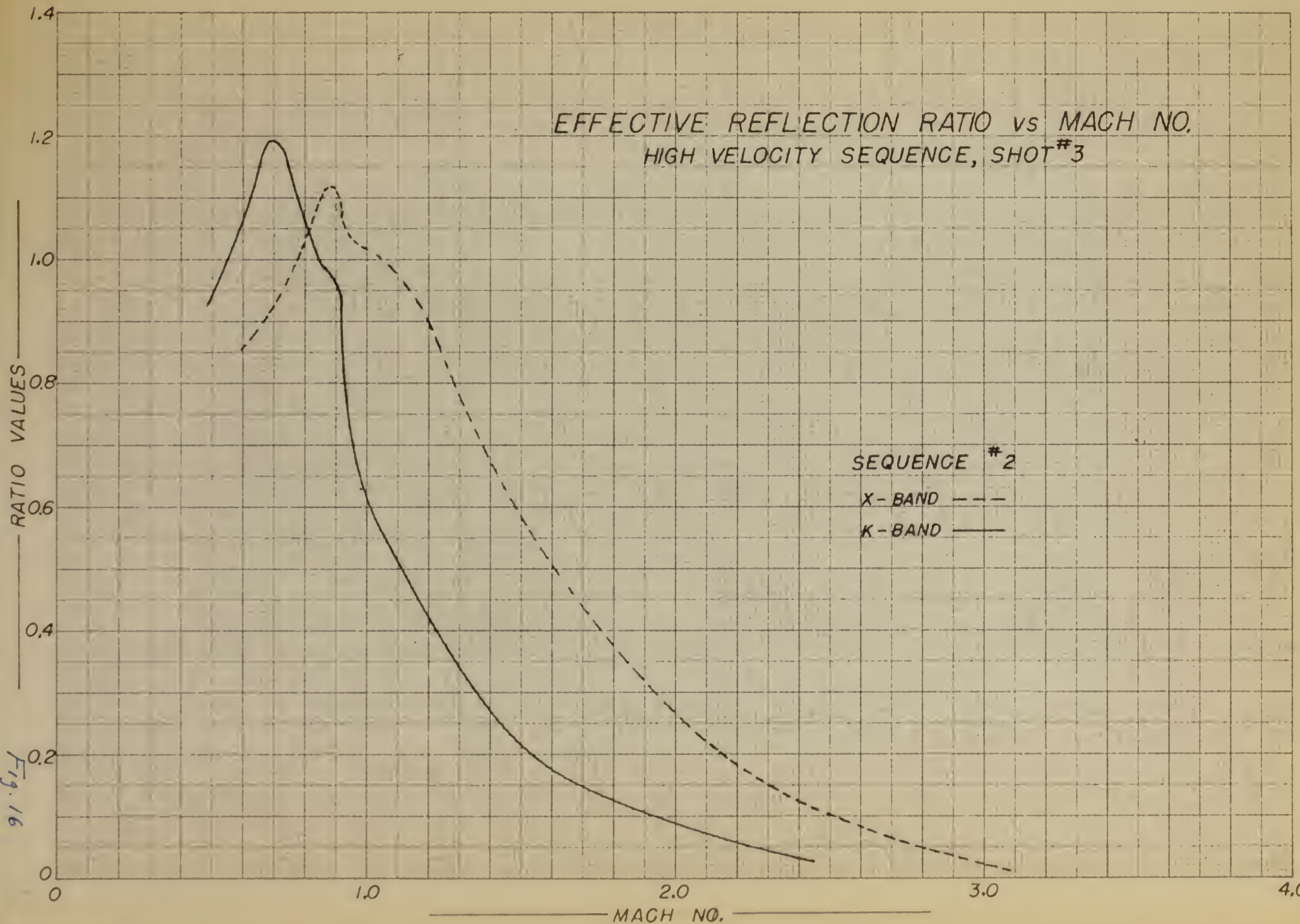

EFFECTIVE REFLECTION RATIO vs MACH NO.
HIGH VELOCITY SEQUENCE, SHOT #3
SEQUENCE #2
X-BAND
K-BAND
RATIO VALUES
MACH NO.
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0
1.0
2.0
3.0
4.0

Fig. 16

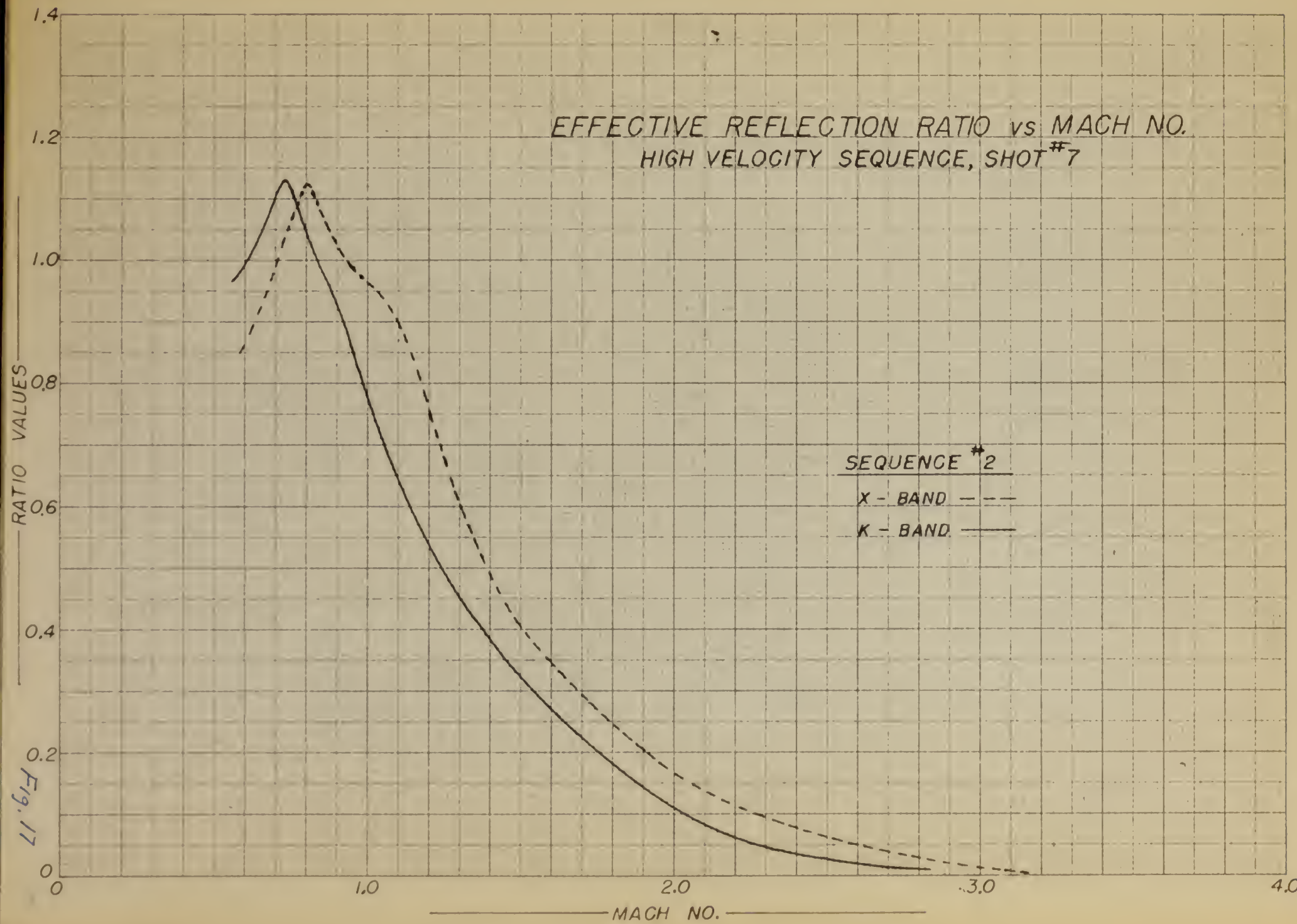

EFFECTIVE REFLECTION RATIO vs MACH NO.
HIGH VELOCITY SEQUENCE, SHOT #7
SEQUENCE #2
X - BAND
K - BAND
RATIO VALUES
MACH NO.
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0
1.0
2.0
3.0
4.0

Fig. 17

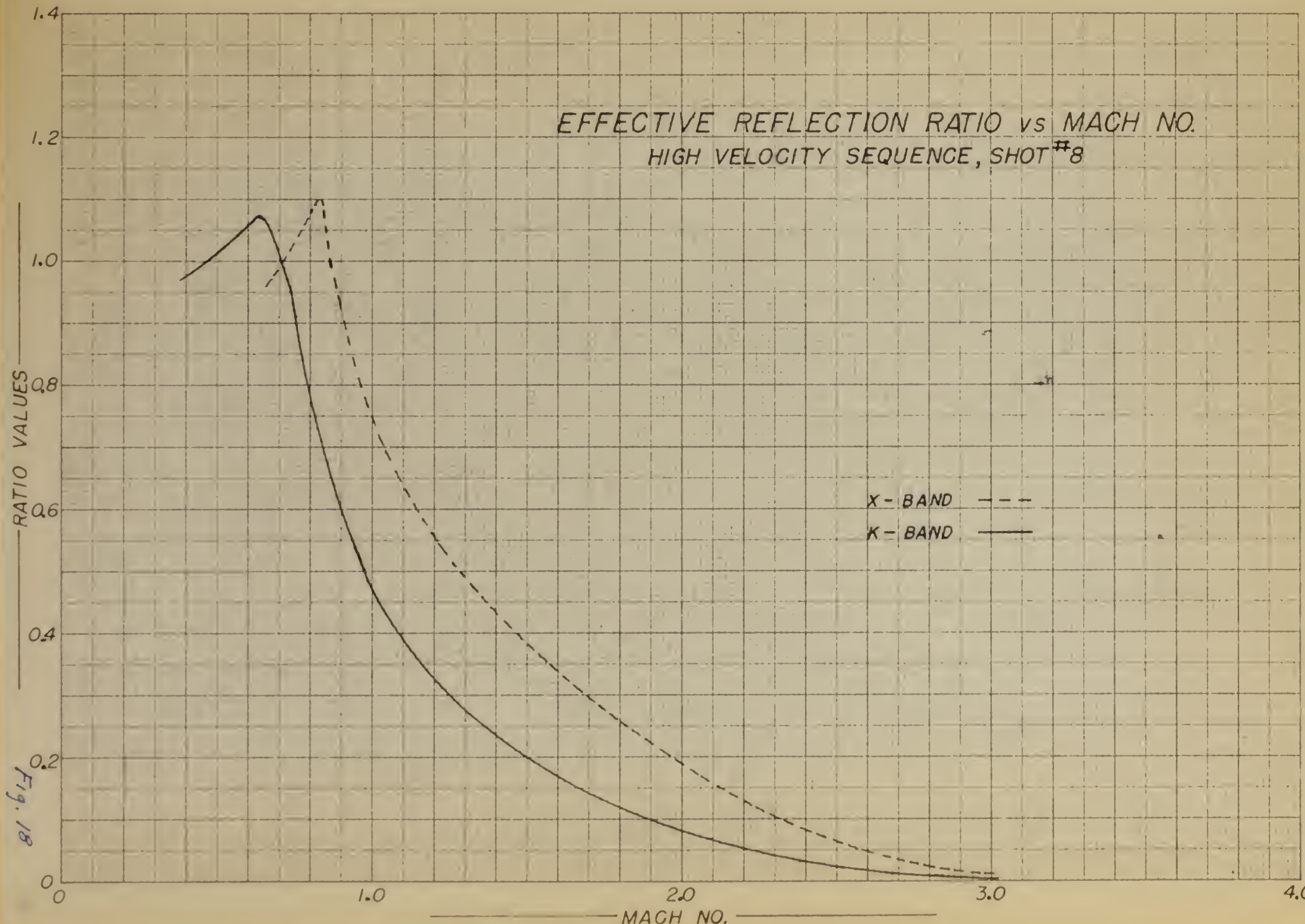

EFFECTIVE REFLECTION RATIO vs MACH NO.
HIGH VELOCITY SEQUENCE, SHOT #8
X - BAND
K - BAND
RATIO VALUES
MACH NO.
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0
1.0
2.0
3.0
4.0

Fig. 18

29e

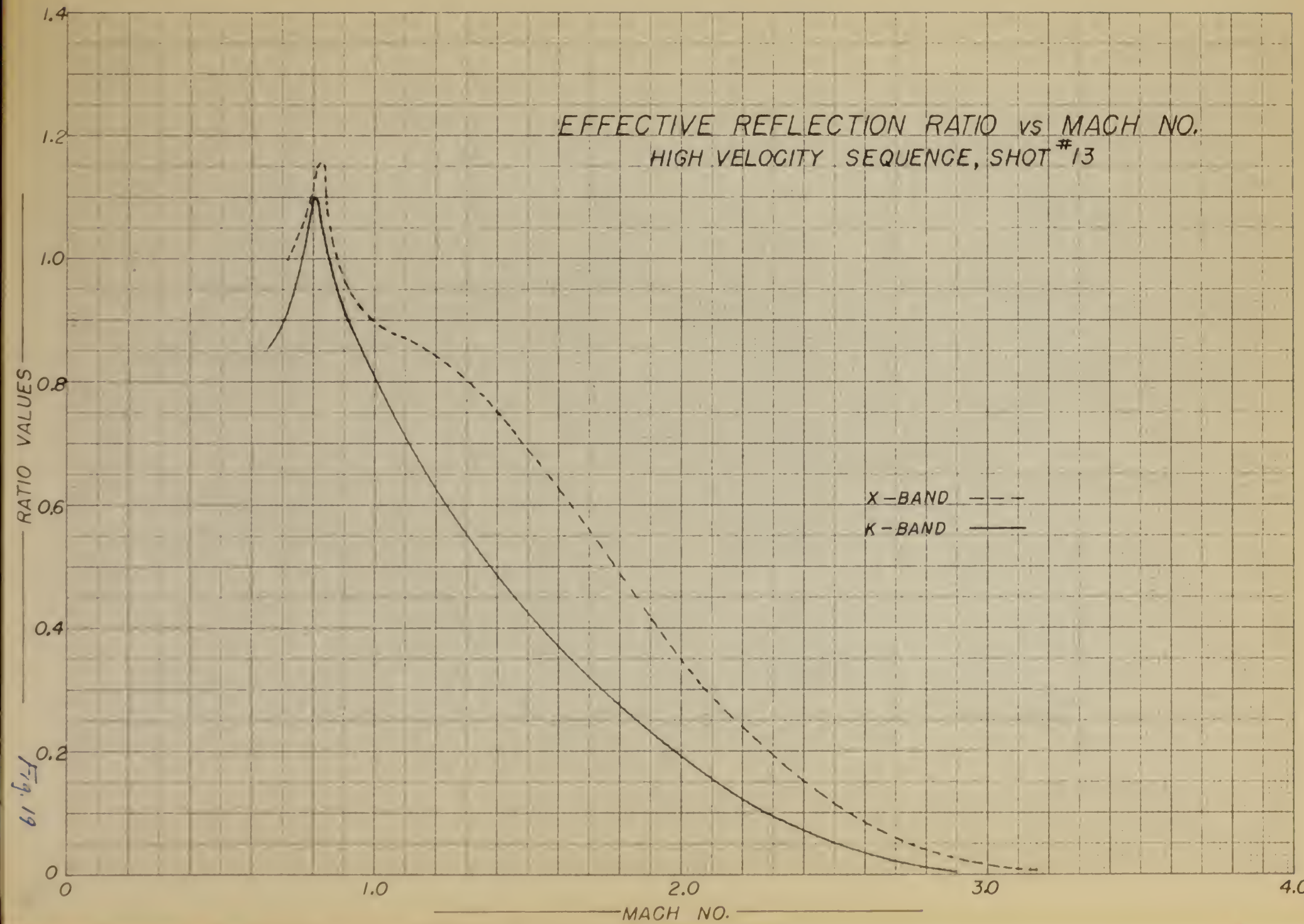

EFFECTIVE REFLECTION RATIO vs MACH NO.
HIGH VELOCITY SEQUENCE, SHOT #13
X-BAND
K-BAND
RATIO VALUES
MACH NO.
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0
1.0
2.0
3.0
4.0

Fig. 19

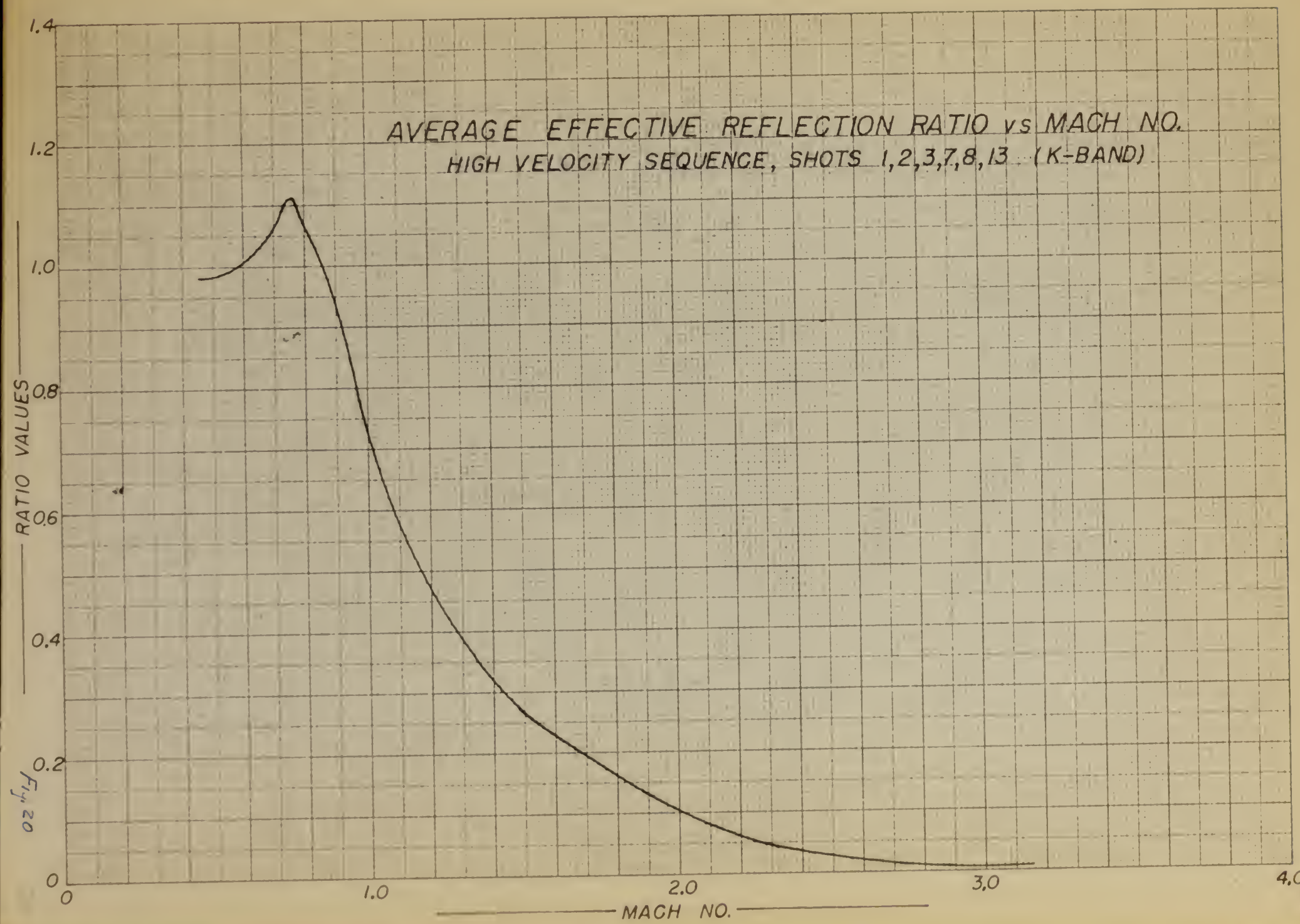

AVERAGE EFFECTIVE REFLECTION RATIO vs MACH NO.
HIGH VELOCITY SEQUENCE, SHOTS 1,2,3,7,8,13 (K-BAND)
RATIO VALUES
MACH NO.
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0
1.0
2.0
3.0
4.0

Fig. 20

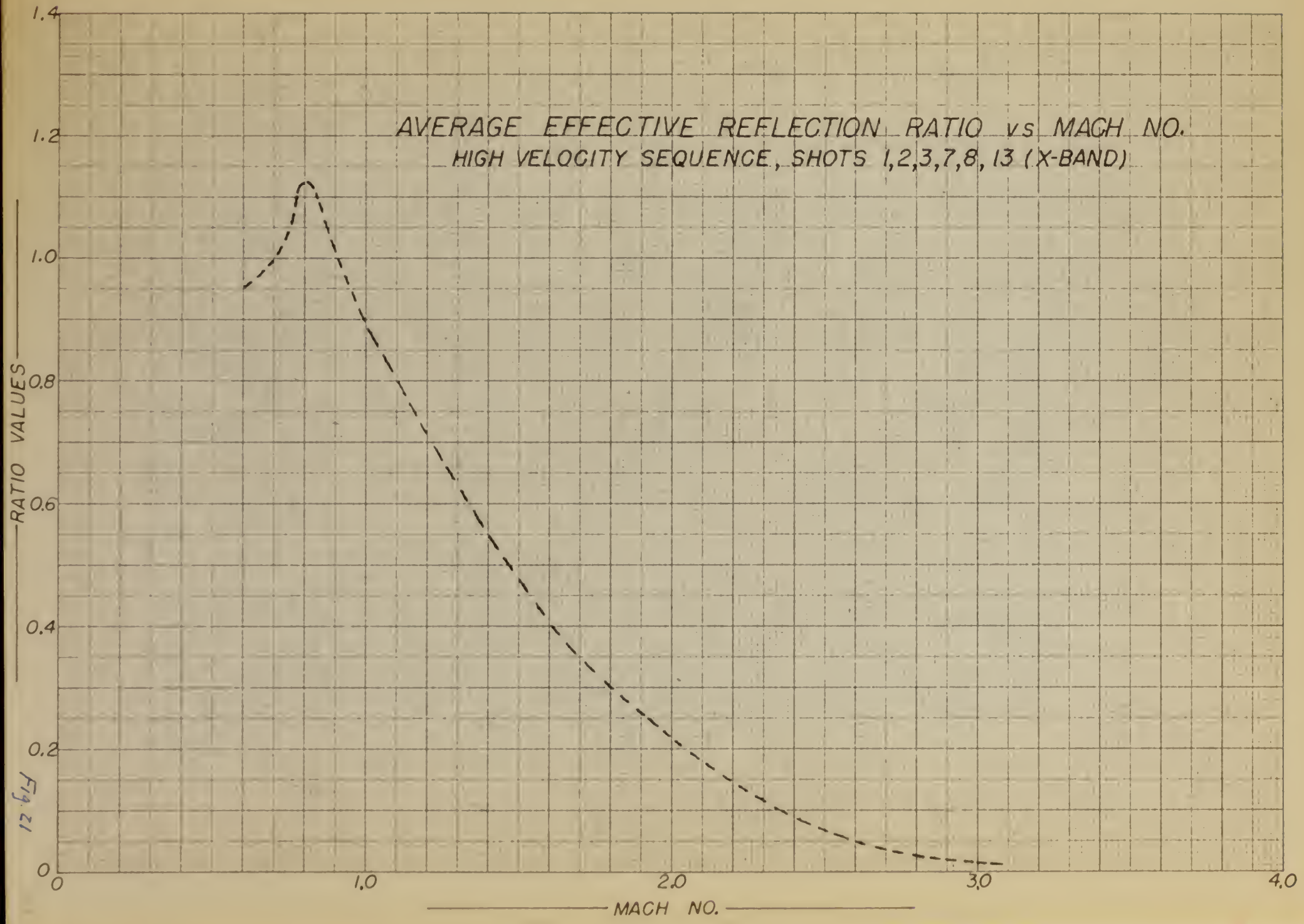

AVERAGE EFFECTIVE REFLECTION RATIO vs MACH NO.
HIGH VELOCITY SEQUENCE, SHOTS 1,2,3,7,8, 13 (X-BAND)
RATIO VALUES
MACH NO.
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0
1.0
2.0
3.0
4.0

Fig. 21

V. Results and Conclusions.

A. Expected Results.

1. Original intent.

The initial attempts that have already been described, section II B 1,2,3, were based on the hope that reflections could be obtained from the shock waves trailing from the bullet, without including any response from the bullet itself. These attempts were unsuccessful, probably because of the short transit time of the bullets across the beam.

2. Modified intent.

With the failure of the above attempts, the proposed basis of the experiment was modified, section II B 4. This modification proposed that as the velocity of the bullet decreased from greater than to less than acoustic velocity, a sudden change (a decrease is to be expected) would occur in the reflection response from the bullet and shock wave. This would occur as the shock wave would weaken and fade out. Further, large surges might be expected in the reflection response as the shock wave approaches the ideal reflection conditions of a corner reflector and in the limit a plane reflector. Thus, if a sudden decrease were experienced, at velocities known to be in the range of Mach number of unity, it might safely be assumed that the lower value is due only to the

reflection, with proper attenuation ascribed to it, from the bullet itself. Thus, the excess over the value predicted by an extrapolation of the attenuation could be directly attributed to the reflection from the shock wave. It was in this direction that the analysis was initially directed. However, as the calculations progressed, it was found that such was not the case. When a comparison was made with values predicted from a simple attenuation relation, instead of an excess in the reflection response, a drop or loss is seen to occur.

3. Difficulties.

The application of a little hindsight to a more detailed study of the problem convinces us that this method of attack would very likely be unsuccessful. There are a few reasons that now become more apparent. If the shock wave boundary is accepted as a reflecting boundary, we are bound by the laws of optical reflection. Hence, we could only expect that the electromagnetic waves would be reflected back to the receiver only in the vicinity of two specific shock wave configurations, both of which are very transitory. One of these is the right angle corner reflection, and the other the plane wave. The plane wave is a weaker wave and its effect as a reflecting boundary might be expected to be small. Thus the transitory corner wave is the only condition for which

reflections might be expected from other than the bullet itself. But even for this lone expected reflection, the problem of the low reflectivity of the shock wave becomes of paramount interest. The low coefficient, of the order of 10^{-5} according to Cowan and Hornig, indicated that the reflection response of a shock would probably be of such a small magnitude as to be indistinguishable.

B. Indicated Results.

1. Effective Reflection Area.

The results, as expressed in the curves of effective reflection area versus Mach number, are at best only approximate indications of the sequence of physical events that are taking place. These curves show that as the bullet, traveling at its maximum velocity, leaves the muzzle, it has a minimum effective reflecting area, even though the reflection response amplitude is at its maximum at this time. As the bullet rises, its velocity decreases and correspondingly, the effective reflection increases and the measured reflection response decreases. The region of major interest for these curves is the so-called transonic zone. Here the effective reflection is seen to rise to a peak, on the order of 10% greater than the indicated unit effective value, and then fall again to the unit value and even lower as the velocity decreases to subsonic values. The general shape of the curve of

effective reflection versus Mach number is consistent enough for each of the shots calculated to permit making the statement that similar effects may be expected for experiments of this nature that might be carried on in the future. A further effect that is consistent throughout the entire analysis is that although the K-band reflection response amplitude is greater than the corresponding X-band response for any particular velocity, the opposite holds for the effective reflection ratio of the two bands.

2. Equation of Motion.

A result of secondary importance that should be included is the solution of the equation of motion of the bullet. The differential equation of motion is accepted as being

$$\frac{d^2x}{dt^2} = -g - b\left(\frac{dx}{dt}\right)^2$$

The solution has been readily determined as

$$x = \frac{1}{2b} \ln \frac{g + b\left(\frac{dx}{dt}\right)^2_{t=0}}{g + b\left(\frac{dx}{dt}\right)^2}$$

and may be further expressed as equation (3") in section IV B 2. The important thing here is that the coefficient of the resistance of the air to the motion of the bullet has been evaluated, for this particular case, as being 8.2×10^{-4} inverse feet. This accuracy is reasonably good, although it is limited because the determination

was made by comparison with the average curve of velocity versus distance.

3. Doppler Effect

A further point of interest is the extent of a Doppler effects at the receiver. Calculations indicate that the effect is negligible, even at maximum velocity of the bullet, the distortion being of the order of magnitude of one- one hundred thousandth of the operating frequency.

C. Conclusions.

1. Summary.

In order to present the conclusion in the simplest possible manner, it would be best to summarize the information presented to this point. The experiment was initiated as an effort to determine the velocity of projectiles with supersonic speeds. The first proposals suggested, in a vague way, measuring the angle of the shock wave associated with the moving object. These proposals were modified to the extent that some efforts were made to measure the reflection of high frequency waves from the shock wave without including reflection from the projectile itself. When these efforts failed, further modifications proposed that a total measurement be made of the reflection from the projectile both with the shock wave and without the shock wave. By this

method it was hoped that the reflection due to the shock wave might be separated from the total and then be analyzed by itself. This, of course, presupposed that the effect of the shock wave would be to increase the total reflection. However, it seemed that the effect of the shock was to reduce the reflection obtained. Fears that this decrease was inherent in the radar system were dispelled when assurance was obtained that the system had a linear response within the range of the experiment. However, to this point there was no definite information that the reduced reflection was due to the shock. At this time, the basis of the experiment was changed from consideration of a direct measure of reflection response to a study of a response ratio, the ratio being a comparison of the measured response with an expected response and called the effective reflection ratio or area. Since the expected response is determined as a function of altitude, and the measured response is recorded as a function of altitude, the value of the ratio is primarily independent of altitude and a function of velocity. Thus, plots of effective reflection ratio versus Mach number are a fairly complete summary of the significant information determined during the analysis.

2. Physical Significance.

The curves of effective reflection versus Mach

number are intimately related to the sequence of physical events. Since comparatively little is known about what actually occurs, it may be possible to add something by a proper interpretation of these curves, or at least confirm previously reported results. Since the events occur in separate regions it would be well to separate the discussions into appropriate sections.

a. Supersonic region.

In this region, no reflection is expected to occur since the shocks do not, for any length of time, attain the conditions required for a reflecting boundary. The point of interest is the very large reduction in effective reflection, the reduction becoming larger for higher Mach numbers, approaching a value on the order of 1% of unit effective reflection area for a Mach number of three. The interpretation ascribed to this effect is that as the bullet travels with supersonic velocities, it has associated with it an attached conical shock wave which precedes the bullet and a low pressure region, which is to be called a vacuum tube, immediately following the bullet. The assumption is made that the length of the vacuum tube is dependent upon the Mach number. It is further assumed that this vacuum tube is bounded by a laminar flow. These assumptions are not to be considered too rigid, but with the lack of more exact information,

they must suffice.

The following hypothesis is now proposed. As the electromagnetic waves proceed toward the target surface, the base of the bullet, they pass through a region of decreasing density, with correspondingly decreasing index of refraction. Accordingly, the electromagnetic beam is partially refracted and reflected away from the normal to the laminar layer and thus deflected from the target area. Since the dimensions involved are probably on the order of a few centimeters or less, not all of the deflected beam bypasses the bullet, and so some contribution may be made to the reflection. Further, part of the beam probably proceeds directly to the target and the remainder of the reflection takes place. For higher Mach numbers, with its longer vacuum tube, the beam approaches the laminar layer at nearly a grazing incident angle, and therefore would, to a large extent, be reflected off away from the bullet. This is one method of accounting for the low reflectivity occuring at the higher Mach numbers.

A second hypothesis is proposed to explain the reduction of effective reflection area. This one is based on an interference phenomenon. It holds that the radar beam is able to proceed directly to the shock front before any major changes occur. The beam is considered

to satisfy optical theory. The wave front is reflected from the shock and interferes with that part of the wave front proceeding to and being reflected from the target area of the bullet. It further proposes that as the Mach number increases a complete destructive interference is approached. It is also suggested that the destructive interference may diminish after a critical Mach number has been passed. Since such a critical condition certainly was not reached during the course of the experiment, the latter suggestion is reduced to an almost unfounded conjecture, and must be neglected. This hypothesis claims, then, that the reduction of effective reflection is strictly a measure of an interference pattern. If this is really the case, then wide new avenues of approach are opened to a study of shock waves, interference phenomena, and their mutual interaction. However, since whatever may be occuring, and however large its time rate of change may be, we are always dealing with a continuous fluid medium and a monochromatic beam. Further, the velocities with which we are concerned are certainly insignificant when compared with the velocity of light. From this it is to be concluded that the series of events with which we are concerned are not functions or results of interference phenomena. But, it must be understood that interference effects are not excluded, merely that they are of minor

importance in this experiment.

b. Transonic Region.

The transonic region is defined by the presence of velocities greater than sonic velocity on the surface of the projectile, even though the projectile itself is traveling at less than sonic velocity. During the transition from supersonic to transonic velocities, the effective reflection increases rapidly to almost the unity value. The physical situation is described by an extension of the former hypothesis; that is to say, that the vacuum tube and its surrounding laminar layer become much shorter in length, and, therefore, the radar beam approaches at a greater incident angle. Thus, less of it is deflected away from the bullet and, consequently, the increase of effective reflection area results. The major interest is centered on the peak in the curves that reaches a maximum in the region of the Mach number of eight-tenths. This peak is surprising in that its appearance was totally unexpected, since no hint was given in conclusions already drawn from the analysis. The peak has a maximum value of effective reflection of roughly 10% greater than that predicted by the inverse fourth power law for a stationary target at a corresponding distance from the source.

The physical picture in the transonic region is that a standing shock is produced on the sides of the missile. These shocks are practically normal to the axis of the bullet, and so are parallel to its base. The interpretation that is made of these peaks is almost identical with the above picture. Some physical phenomenon is occuring which increases the reflection to a greater value than that expected. Since it is unlikely that another energy source has been tapped on the bullet and is transmitting to the ground, another explanation is required. The most logical is that the target area has been enlarged. Thus the conclusion is reached that the standing shock is contributing to the increase. This conclusion, however, contradicts the previous assumption that no response is to be expected from a plane wave, But this is well-founded in other experimental data and results, while the former is not. It may now be stated that it is possible to detect shock waves by high frequency electromagnetic waves. The process by which the shocks are detected, at least in the transonic region, leads to some interesting speculations. If the coefficient of reflectivity of the shock front could be determined within reasonable limits of accuracy, it would be possible to calculate the height of the standing shock above the missile profile. Or what is perhaps more likely, with the wealth of transonic wind tunnels available today, that the height of the shock

may be measured almost directly, and thus the coefficient of reflectivity be calculated, leading to a fairly accurate result. It would be interesting to compare this value with that of Cowan and Hornig, roughly 10^{-5}.

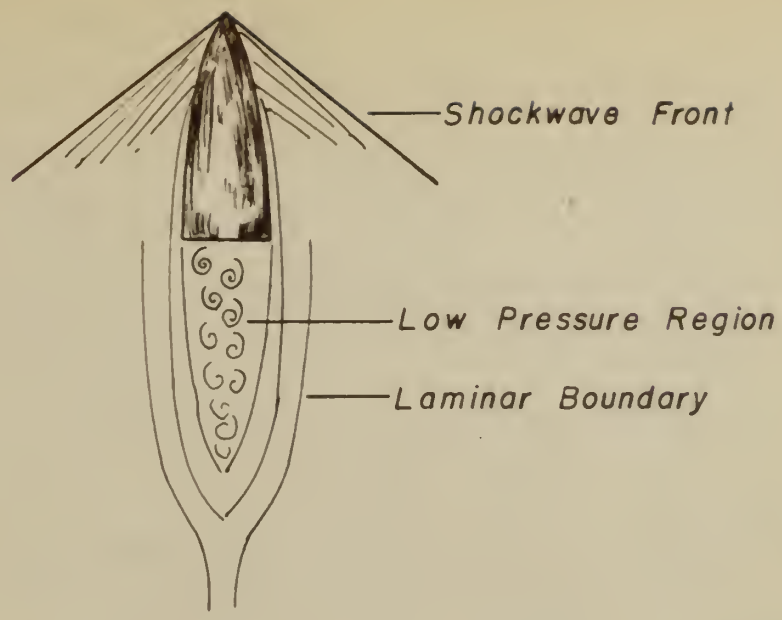
c. Subsonic Region.

In the subsonic region, the curves fall off somewhat from an expected value of unity. However, it would be inadvisable to attempt any specific conclusions about events in this region, since data measurements in the region of Mach number six-tenths and less are suspect with regard to accuracy. At best it seems safe to say that the predominating influence is the stable laminar flow pattern. However, gradients of pressure, density, and optical index of refraction do not have the steep characteristics that typify flow in higher velocity ranges. Therefore, some reduction in effective response may be expected, but to a much lesser extent than in the higher ranges.

d. Hidden Effects.

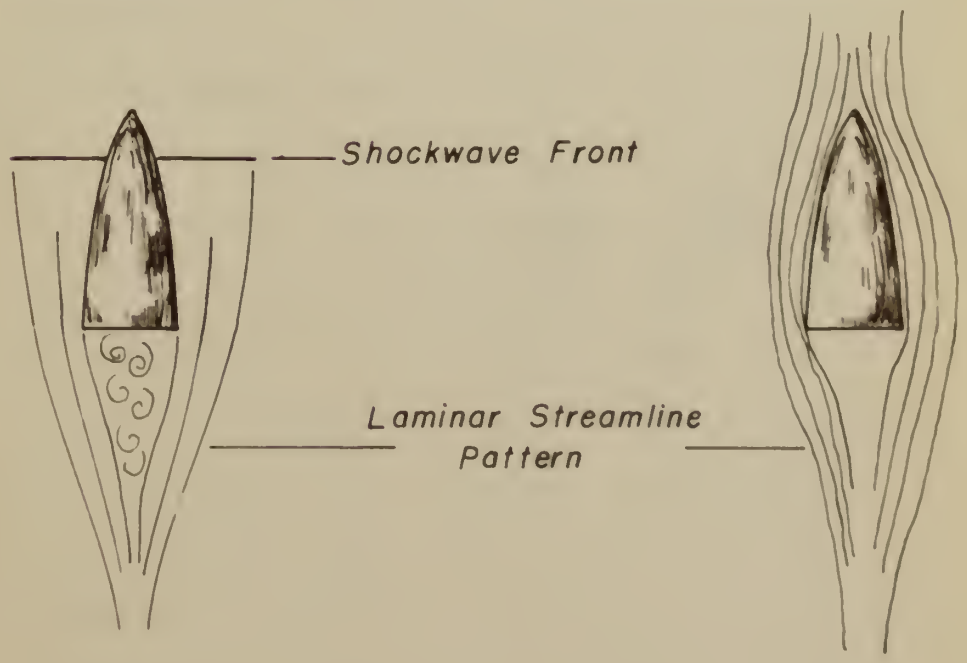
It should be noted that since the constant of proportionality was determined from data for Mach less than unity, some of the effects that occur in the subsonic range will not become apparent but will be carried throughout as a hidden constant. The problem inherent in an analysis of this nature may also be studied as a part of further more detailed investigations.

(1) SUPERSONIC



(2) TRANSONIC

(3) SUBSONIC



SKETCH OF ASSUMED FLOW PATTERNS
(FOR HIGH VELOCITY BULLETS)

Fig. 22

D. Additional Comments.

It is still necessary to give a satisfactory justification for the acceptance of the assumption that the bullet acts as a reflector and not as a scattering particle. It will be recalled that conditions have been defined for both types of action. If the dimensions of the object are large compared to the wave length of the incident radiation, then the object acts as a reflector. If, on the other hand, the dimensions of the object are small compared to the incident wave length, the object acts as a scattering particle. However, for this particular problem, the dimensions are of the same order of magnitude. Thus, there is no indication as to which action is taking place. For convenience, the analysis was based on the assumption of reflection, which means that attenuation was according to an inverse fourth power law. There are two main check points for this assumption. One is the plot for both frequencies, of the average of measured and predicted responses versus altitude to the fourth power, and its companion curve with the same ordinate versus the inverse fourth power. The other is the log-log plot for 3 shots on the K-band of the response versus altitude. In all these curves, there is very close agreement, in the subsonic region, between values of the measured response curves and those of the predicted

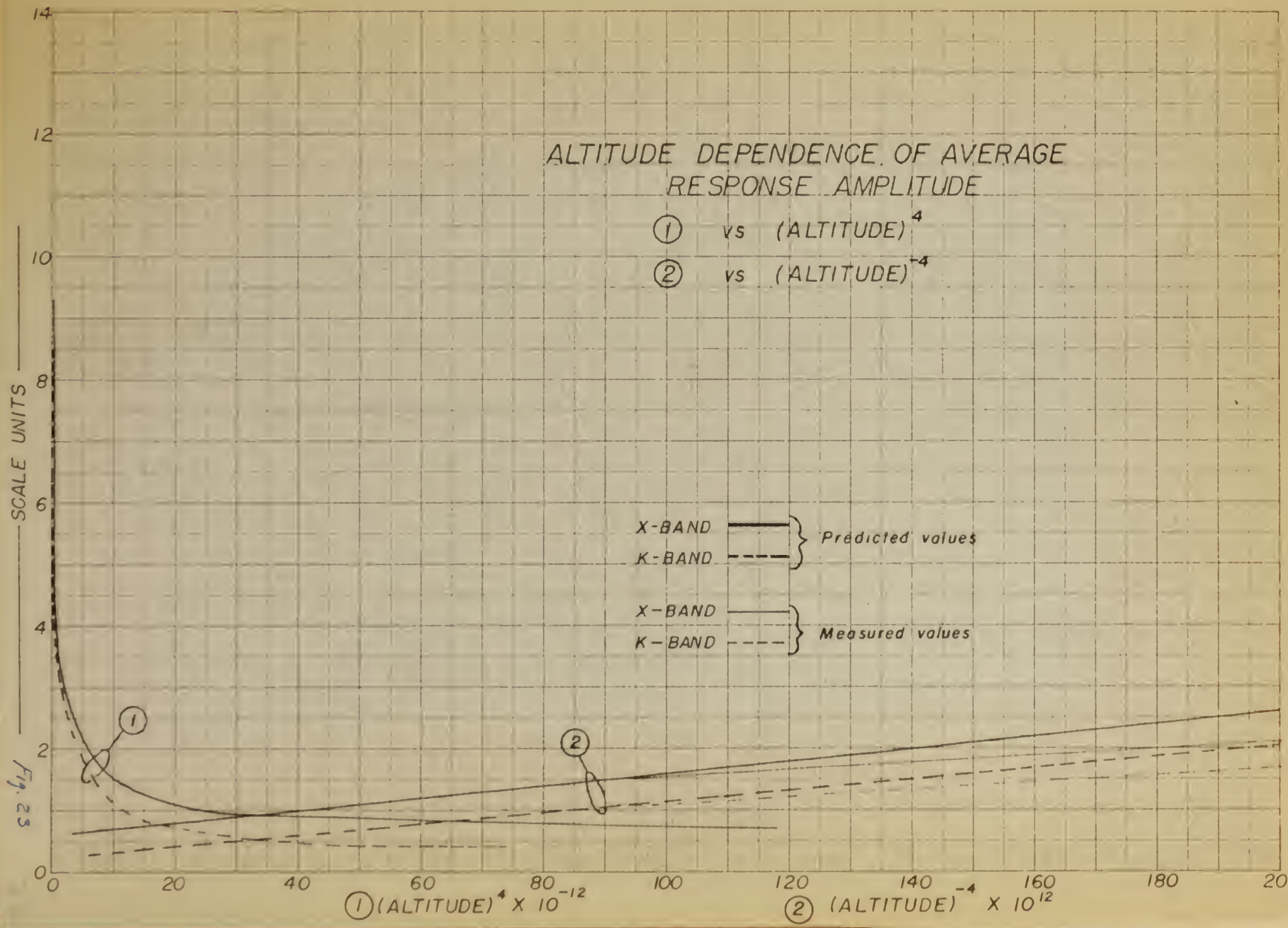

ALTITUDE DEPENDENCE OF AVERAGE RESPONSE AMPLITUDE
① vs (ALTITUDE)^4
② vs (ALTITUDE)^-4
X-BAND
K-BAND
Predicted values
X-BAND
K-BAND
Measured values
SCALE UNITS
0
2
4
6
8
10
12
14
0
20
40
60
80
100
120
140
160
180
200
① (ALTITUDE)^4 X 10^-12
② (ALTITUDE)^-4 X 10^12

Fig. 23

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response curves. This constancy of agreement is such that the assumption of the reflector action can be accepted. The importance of this agreement is not that we have chosen the proper assumption, but that new limits have been put upon the distinguishing conditions for reflectors and scatterers. According to Stratton[3], these intermediate situations are too involved to permit satisfactory mathematical analyses. Perhaps, now that experimental evidence is available, the mathematical derivation may be extended to a satisfactory conclusion. Summing up this conclusion, it may be said that the phenomenon of reflection, as opposed to scattering, can occur if the dimensions of the target are of the order of, or greater than, the half wave length of the incident electromagnetic radiation. Symbollically, this information may be expressed as

target size	wave length	attenuation dependence		source
$\rho \gg$	λ	d^{-2}	reflection	known[3]
$\rho \ll$	λ	d^{-4}	scattering	known[3]
$\rho \approx$	$\frac{\lambda}{2}$	d^{-2}	reflection	experimental result.

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ABSTRACT

The paper has its origin in proposals to attempt the determination of the velocity of supersonic missiles by reflecting high frequency electromagnetic waves from the shock waves known to be associated with high speed bodies. An experimental technique was evolved utilizing some radar equipment of the Bell Telephone Laboratories. This equipment operated at frequencies on the order of 10,000 and 30,000 megacycles with wave lengths of 3.2cm. and 1.25 cm., respectively, the pulses being interlaced and the antenna vertically orientated. Two types of .22 calibre bullets were used as the high speed missiles, a "long rifle" with a muzzle velocity of 2,200 feet per second, and a high velocity with a muzzle velocity of 4,400 feet per second. The bullets were fired vertically along the center line between the antennas. The reflected response from the bullets was impressed on a double trace oscilloscope tube, the upper trace for the 3.2 cm. wave and the lower trace for the 1.25 cm. wave. In order to have a complete record of the events pictured on the tube, a set of photographs were taken by a 16mm. motion picture camera. A total of 16 long rifle and 13 high velocity bullets were fired.

The transcription of the information recorded on the

film was a secondary problem but nonetheless essential to the analysis of the experimental data. The film strip was marked so that specific frame sequences could be chosen for projection. The image was first projected onto a ground glass screen, an integral part of the projector, and later was focused on a wall about five feet away. To assist in reading the projected image, a mil scale was included in the light path for the ground glass image, but for the larger wall image a sheet of 10 square to the half inch cross-section paper was tacked to the wall. The following information was recorded: 1) frame number, which sufficed as a time measure; 2) distance along the trace of the response peak from the origin; and 3) amplitude of the response peak for both the frequency ranges employed.

The analysis of the above data was carried on under the assumption that the bullet does not act as a scattering particle with regard to the incident radiation, but acts as normally would an optical reflector. Thus, predictions may be made for comparison of attenuation on the basis of the inverse fourth power of target distance. In order to provide a basis for conclusions with regard to the reflection from the shock wave, it becomes essential to determine a velocity dependence for the reflection response. This is done in a roundabout fashion. Velocity-distance

curves are obtained using time as a parameter. It is then possible to determine reflection response-velocity relations using the distance as a parameter. But this result is apparently inconclusive. In hopes of determining more specific information, a comparative ratio, called the effective reflection area, was evaluated. This ratio is defined by comparing the measured reflection response for the missile with the expected reflection for the same object as a stationary target at the same altitude. The curves of effective reflection area versus Mach number summarize the main results of the analysis. Their general shape, which is consistent for each of the shots evaluated, may be discussed in three sections: 1) subsonic; 2) transonic; and 3) supersonic. In the subsonic region, the values of the effective reflection area are on the order of unity, although they are somewhat lower than this. In the transonic region, the curves rise to a slight peak, roughly 10% greater than unity, and come down again as the supersonic region is approached. In the supersonic region, the values of the effective area fall off sharply, attaining a value of roughly 1% at a Mach number of three. The fall is smooth and continuous and approaches a low value horizontal assymptote.

The physical significance attached to these curves is also best discussed in three corresponding sections, but

in reverse order. The supersonic reduction leads to the conclusion that the bullet is followed by a long thin low pressure region which is probably bounded by a laminar layer. The radar beam, then, is to a large extent deflected away from the target area by optical reflection and refraction; therefore, little of it is allowed to reach the target area, the base of the bullet. When the velocity of the bullet has decreased to the transonic range, the prominent feature is the standing shock generated on the side of the bullet. The peak in the response curve is attributed to reflection coming from the transonic shock. The effect of this shock is thus specified as increasing the target area. As the velocity becomes subsonic this shock disappears, and the effective reflection falls off accordingly.

A second hypothesis which attempts to explain the results in terms of an interference phenomenon is also proposed. However, the explanation is unsatisfactory over the entire range of events in the experiment, and therefore, is reduced to minor importance.

Some calculations, of secondary importance, which were carried out are the Doppler effect at the receiver and the equation of motion of a projectile moving upward with an initial velocity. The Doppler calculations show that this effect is negligible. The calculations for

the equation of motion are interesting, for the constant coefficient of the resistance of the air has been evaluated. By a trial and error method of comparing the derived curve with the average curve, the value of the coefficient was found to be 8.2×10^{-4} inverse feet.

The substantiation of the inverse fourth power assumption for the attenuation of the reflection response amplitude indicates a further important conclusion. A target will act as a reflector, as opposed to a scattering particle, when its dimensions are of the order of, or greater than, the half wave length of incident electromagnetic radiation. This extends the known limits of optical reflection to targets of dimensions at least as small as the order of magnitude of the incident half wave length.

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